Homework 4 Solutions

Physics 406.

1. O chap. 4 Q1.

Delocalized electrons in solids respond to the application of an electric field whereas their localized (core) counterparts do not.

A measurement of the current density can be used to determine the concentration of current carriers or delocalized electrons

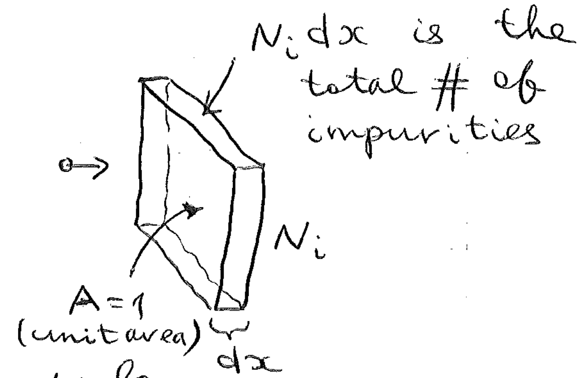
2. O chap. 4 Q2

- In a plasma the constituents are charged (not usually the case for a gas) similar to the situation for conduction electrons

- Plasma is higher in density than is ordinary gas – for free electron gas $N \sim 10^{29}$ electrons / m^3 whereas for ordinary gas $N \sim 10^{25}$ molecules / m^3.

3. Omar P. 2
Ch. 4

Recall that MFP $\ell = \tau v$,
such that $\frac{dx}{\ell}$ is the probability to have a collision in going from x to $x+dx$.



Now, consider a particle which travels distance dx through material w/ N_i impurities per unit volume. Each impurity has cross-section (eff. collision area) σ_i.
Then total area covered by the scatterers is $\sigma_i N_i\,dx$ (recall that $A=1$), and the prob. to have a collision is also $\sigma_i N_i\,dx$.

Thus $\sigma_i N_i\,dx = \frac{dx}{\ell}$, or $\ell = \frac{1}{N_i \sigma_i}$.

4. $\frac{v_d}{v_F}$ for Cu wire w/ 10 Amp / mm^2

From Table 4.1 in Omar we have for Cu

$v_F = 1.6 \times 10^6$ m/sec

$N = 8.45 \times 10^{28}$ e/m^3

$$j = \sigma E = N e v_d \qquad j = 10\,A / 1\,mm^2 = 10\,C/s / 10^{-6}\,m^2$$

$$\Downarrow$$

$$v_d = \frac{j}{Ne} = \frac{(10\,\cancel{C}/s) / 10^{-6}\,m^2}{(8.4 \times 10^{28}\,\cancel{e}/m^3)(1.6 \times 10^{-19}\,\cancel{C}/\cancel{e})}$$

$$= \frac{10^7}{10^9} \times \frac{1}{8.4 \times 1.6} \quad m/s$$

$$= 10^{-2} \times .074 \quad m/s$$

$$v_d = 7.4 \times 10^{-4} \quad m/s.$$

$$\Downarrow$$

$$\frac{v_d}{v_F} = \frac{7.4 \times 10^{-4}\ m/s}{1.6 \times 10^{6}\ m/s}$$

$$= \frac{7.4}{1.6} \times 10^{-10} = 4.63 \times 10^{-10}$$

$$\boxed{\frac{v_d}{v_F} = 4.63 \times 10^{-10}}$$

!

5. O chapter 4, Problem 6.

T/T_F for $T = 300$ for Cu, Na and Ag.

From Table 4.1 in Omar we have

Element	E_F (eV)
Cu	7.0
Na	3.1
Ag	5.5.

$$E_F = k_B T_F \Rightarrow T_F = E_F / k_B .$$

$$k_B = 1.4 \times 10^{-23} \ J/K = 8.6 \times 10^{-5} \ eV/K$$

Element	T_F	$T/T_F \big|_{T=300}$
Cu	8×10^4	3.8×10^{-3}.
Na	3.6×10^4	8.3×10^{-3}.
Ag	6.4×10^4	4.6×10^{-3}.

6. O Chapter 4 Problem 7.

Fraction of electrons excited above Fermi level at $T \sim 300\,K$ $\sim$ $\dfrac{k_B T}{E_F}$

Cu $\Rightarrow$ $E_F = 7.0\,eV$ $\Rightarrow$ $f = \dfrac{(8.6 \times 10^{-5})(300)}{7}$

$\Downarrow$

$f_{Cu} = 3.7 \times 10^{-3}$

Na $\Rightarrow$ $E_F = 3.1\,eV$ $\quad f_{Na} = 8.3 \times 10^{-3}$

7. Kittel chap. 6 problem 4.

a) N_e = # electrons

$N_p \sim \dfrac{M_\odot}{m_p}$

$\downarrow$

protons in sun

$$= \frac{2 \times 10^{33}\,g}{1.7 \times 10^{-24}\,g} \sim 10^{57}$$

Let us assume that there are roughly the same number of e^-'s and p's

$$\Downarrow$$

$$\mathcal{E}_F = \frac{\hbar^2}{2m} \left(3\pi^2 n \right)^{2/3}$$

where

$$n = \frac{N_e}{V} \qquad \text{where} \quad V = \frac{4}{3} \pi R_s^3.$$

$$= \frac{4}{3} \pi \left(2 \times 10^9 \right)^3$$

$$\Downarrow$$

$$\sim 3 \times 10^{28}$$

$$n \sim \frac{10^{57}}{3 \times 10^{28}} \sim 3 \times 10^{28} \text{ electrons/cm}^3.$$

$$\Downarrow$$

$$\mathcal{E}_F \sim \frac{\left(10^{-27}\right)^2}{2\left(9 \times 10^{-28}\right)} \left(3\pi^2 \left(3 \times 10^{28} \right) \right)^{2/3}$$

$$\sim \frac{1}{2}\left(10^{-27}\right)\left(10^{20}\right) \sim 5 \times 10^{-6} \text{ ergs}$$

$$\varepsilon_F \sim 5 \times 10^{-6} \text{ ergs} \times \frac{1 \text{ eV}}{1.6 \times 10^{-12} \text{ ergs}}$$

$$\Downarrow$$

$$\boxed{\varepsilon_F = 3.6 \times 10^{4} \text{ eV}}$$

b) k_F is not affected by relativity

In 3d we determine k_F

$$\Downarrow$$

$$N = \frac{3 \cdot \frac{4}{3} \pi k_F^3}{(2\pi)^3 / V} \Rightarrow k_F \sim \left(\frac{N}{V}\right)^{1/3}$$

$$\Downarrow$$

In the relativistic limit

$$\boxed{\varepsilon_F = \hbar k_F c \sim \hbar c \left(\frac{N}{V}\right)^{1/3}}$$

c) Now $\tilde{R}_s = 10 \text{ km} = 10^6 \text{ cm}$

$(R_s = 2 \times 10^9 \text{ cm})$

$$n \sim 3 \times 10^{28} \frac{e}{\text{cm}^3} \times \frac{(2 \times 10^9)^3}{10^{18}}$$

$$\sim 2.4 \times 10^{38} \, e/\text{cm}^3.$$

$$\varepsilon_F \sim \hbar c n^{1/3} \sim (10^{-27})(3 \times 10^{10})(10^{13})$$

$$\sim 2 \times 10^{-4} \text{ erg} \times \frac{1 \text{ eV}}{1.6 \times 10^{-12} \text{ erg}}$$

$$\Downarrow$$

$$\boxed{\varepsilon_F \sim 10^8 \text{ eV}}$$

relativistic

$(m_e/c^2 \sim .51 \times 10^6 \text{ eV})$

9. Kittel chapter 6 Problem 5.

$$\varepsilon_F = \frac{\hbar^2}{2m} (3\pi^2 n)^{2/3}$$

$$\rho = .081 \ g/cm^3$$

$$\frac{\# \text{ moles}}{cm^3} = \frac{1}{3} \ 81 \times 10^{-3}$$

$$= 27 \times 10^{-3} = 2.7 \times 10^{-2} \ \frac{\text{moles}}{cm^3}.$$

$$n_H = \text{concentration of atoms} = 2.7 \times 10^{-2} \ \frac{\text{moles}}{cm^3} \times 6 \times 10^{23} \frac{\text{atoms}}{\text{moles}}$$

$$= 1.6 \times 10^{22} \text{ atoms}/cm^3.$$

$$m_H = (3)\, m_p = (3)(1.6 \times 10^{-24} \ g)$$

$$\sim 5 \times 10^{-24} \ g.$$

$$\varepsilon_F \sim \frac{(10^{-27})^2}{2.5 \times 10^{-24}} \quad \left(3 \cdot \pi^2 \, (1.6 \times 10^{22})\right)^{2/3} \cdot$$

$$\sim \underbrace{\frac{10^{-54}}{10^{-23}}}_{} \quad \left[(3\pi^2)(16)(10^{21}) \right]^{2/3}$$

$$\sim 10^{-31} \cdot \left[[30][16] \right]^{2/3} \cdot 10^{14}$$

$$\boxed{\varepsilon_F \sim 6 \times 10^{-16} \text{ erg}}$$

$$\boxed{T_F = \frac{\varepsilon_F}{k_B} \sim \frac{6 \times 10^{-16} \text{ erg}}{1.4 \times 10^{-16} \text{ erg}/K} \sim 4.29 \text{ K}}$$

10. • Cooling and equilibration of gases at low temperature achieved through head-on "s-wave" collisions - not possible for fermions, due to Pauli exclusion principle, which makes it challenging to achieve low temperature in equilibrium

• Successful cooling of gas of fermionic atoms using s-wave collisions of mixtures of atoms that are in different states

• Jin's group used atoms in two distinct spin states

• Hulet's group used mixture of isotopes

• Evaporative cooling used for these mixtures to achieve quantum degeneracy via collisions