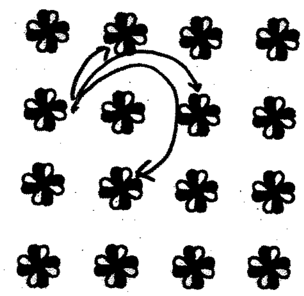
Homework 1 Solutions

Physics 406

1.

a)

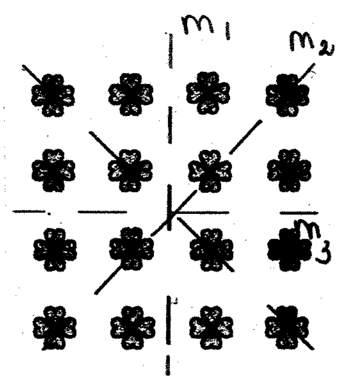


a

4 elements in the symmetry group:

- Identity, 90°, 180° and 270° rotations

b)



b

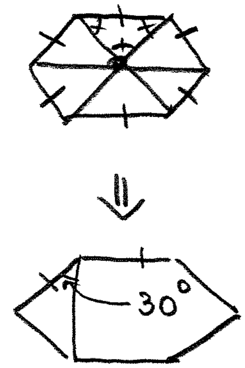
m_4

8 elements in all

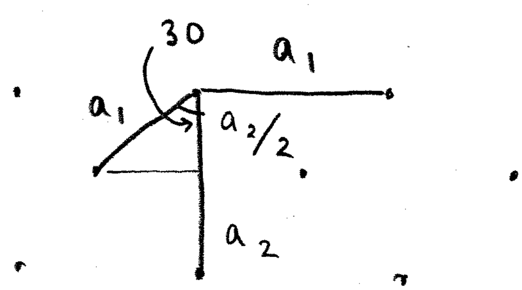
- Same 4 as in a)
- 4 mirror symmetries: m_1, m_2, m_3 and m_4

c) The polygon has the same symmetries as the crystal in b) $\Rightarrow$ this point group is denoted by 4mm.

2 Please recall key features of a hexagonal lattice



Therefore, in order for a centered rectangular lattice to be hexagonal, we must have



$$a_1 \cos 30 = \frac{a_2}{2}$$

$$a_1 \frac{\sqrt{3}}{2} = \frac{a_2}{2} \Rightarrow \boxed{a_2 = \sqrt{3}\, a_1}$$

3. O 1:1

The primitive basis vectors of the lattice are

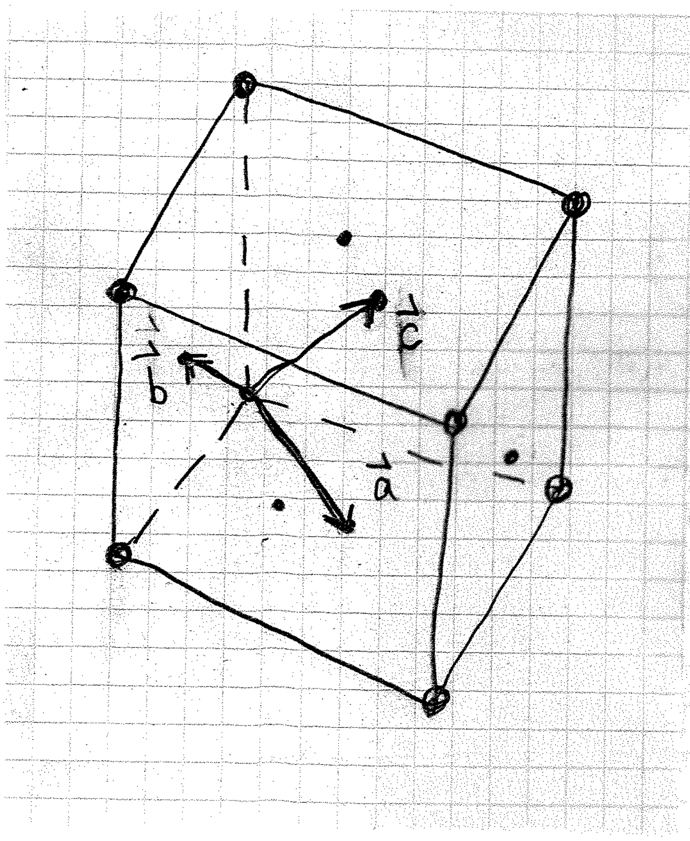
$$\vec{a} = \frac{a}{2}(\hat{i} + \hat{j})$$

$$\vec{b} = \frac{a}{2}(\hat{j} + \hat{k})$$

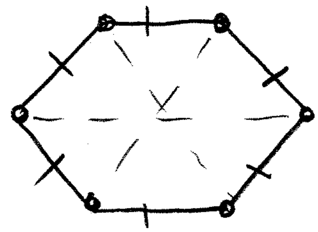
$$\vec{c} = \frac{a}{2}(\hat{k} + \hat{i})$$

$$\Downarrow$$

Bravais lattice is face-centered cubic

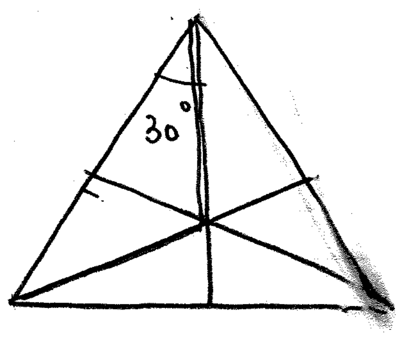


4. Let us represent the centers of the spheres (of radius a) as points. Then a bottom layer hexagonal placquette looks like

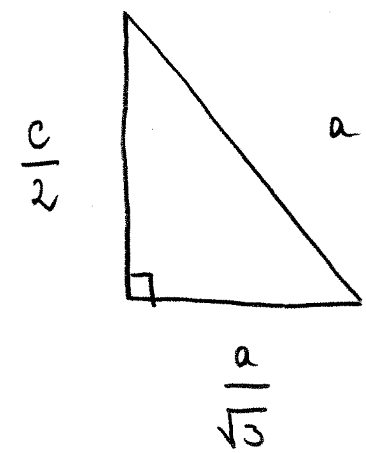


where we note that all Δs are equilateral.

There will be an intervening layer in the hcp such for each triangle of atoms in the bottom layer an atom resides at a position equidistant from each triangular vertexe



In order to solve for e, we study the right triangle



$$\left(\frac{c}{2}\right)^2 + \frac{(a)^2}{3} = a^2$$

$$\left(\frac{c}{2}\right)^2 = \frac{2}{3}\, a^2$$

$$\left(\frac{c}{a}\right)^2 = \frac{8}{3} \Rightarrow \frac{e}{a} = \sqrt{\frac{8}{3}}$$

$$\frac{e}{a} = 1.633.$$

5. In order to determine the x- y- intercepts of these planes, we must invert the indices (thus running the Miller index prescription backwards).

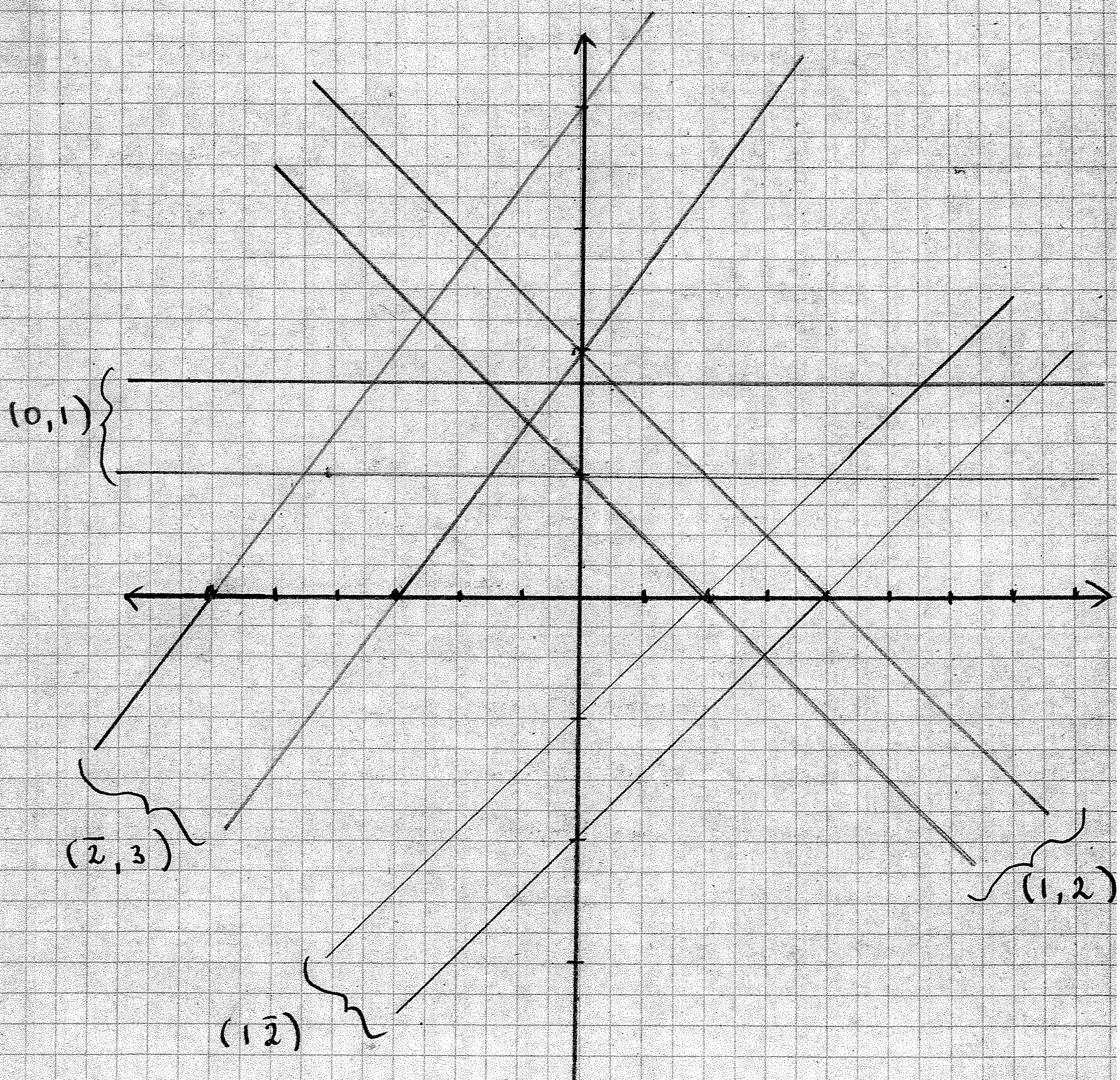
$$(0\ 1) \rightarrow (0\ 1)$$

$$(1\ 2) \rightarrow (1\ \tfrac{1}{2}) \rightarrow (2\ 1)$$

$$(\bar{2}\ 3) \rightarrow (\tfrac{\bar{1}}{2}\ \tfrac{1}{3}) \rightarrow (\bar{3}\ 2)$$

$$(1\ \bar{2}) \rightarrow (1\ \tfrac{\bar{1}}{2}) \rightarrow (2\ \bar{1})$$

The planes are then displayed for a rectangular lattice ($a_2 = 2a_1$) on the next page.


(0,1)
($\bar{2}$, 3)
(1, 2)
(1 $\bar{2}$)