# HW#5 solutions

Physics 406

(1.) $\theta$ Ch. 4, Q. 5

With thermal conduction, $\bar{e}$'s at one end of the sample have more energy than $\bar{e}$'s at the other end. More energetic $\bar{e}$'s diffuse down the T gradient, carrying a net energy flux. On average, there is no particle or charge buildup which would be quite unfavorable energetically.

(2.) $\theta$ Ch. 4, Q. 8

The Hall constant is defined as

$$R_H \equiv \frac{E_H}{J_x B}$$

, where $E_H$ is the Hall field, B is the external magnetic field, and $J_x$ is the current density. Since $J_x \sim N$, ($\bar{e}$ conc'n)

$$R_H \sim \frac{1}{N}.$$

(3) O. Ch. 4, Q. 9

positive charge carriers mean that $J_x = +Nev_x$

$$\Downarrow$$

$$R_H = \frac{E_H}{J_x B} \sim +\frac{1}{e} > 0$$

(4.) $\theta$ Ch. 5, Q. 1

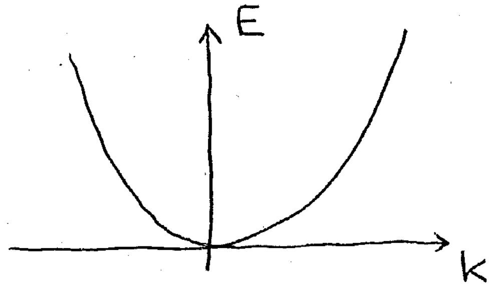
Here we must distinguish between core (localized) and valence (delocalized) $\bar{e}$'s. The combination of the core $\bar{e}$'s & the ions produces weak periodic pseudopotential. In other words, core $\bar{e}$'s "screen" the ions. Bloch's theorem states that the periodic nature of the resulting potential leads to valence $\bar{e}$'s with propagating (delocalized) wavefunctions. Thus there is no paradox - rather there are 2 types of $\bar{e}$'s.

(5.) $\theta$ Ch. 5, Q. 2

For a truly free $\bar{e}$,

$$\begin{cases} \psi_k^{(0)} = \frac{1}{\sqrt{L}} e^{ikx}, & \leftarrow \text{(1d)} \\ E_k^{(0)} = \frac{\hbar^2 k^2}{2 m_0} \end{cases}$$

This leads to a single dispersion curve:




"Cutting & pasting" is not justifiable here because there is no periodicity of the lattice:

$$E_k = E_{k+G}, \quad G = \frac{2\pi n}{a}, \quad n = 0, \pm 1, \pm 2, \ldots$$

Thus the difference between empty lattice & free space is that we impose symmetry properties in k-space (even though there is no potential) in the former case. These symmetries follow from the translational symmetry of the real lattice.

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(6.) θ Ch. 4, Pr. 10

$$\nu_c = \frac{\omega_c}{2\pi} = 2.8\,B \text{ GHz} \quad \text{for } m^* = m_0,$$

$B$ in kG; $m_0$ the free $\bar{e}$ mass.

Then $\nu_c = 24$ GHz gives

$$B = \frac{24}{2.8} \approx 8.6 \text{ kG}.$$

7. θ Ch. 5, Pr. 14

a) Free $\bar{e}$ model:

$$n = 2\,\frac{1}{(2\pi)^3}\,\frac{4}{3}\pi k_F^3 = \frac{1}{3\pi^2}k_F^3, \text{ or}$$

2: spin

$$k_F = (3\pi^2 n)^{1/3}.$$

b) Fermi sphere will touch the face of the 1st BZ when

$k_F = k_i$, where

$\vec{k}_i = \frac{1}{2}\vec{a}$ for fcc, and (or $\frac{1}{2}\vec{b}$, $\frac{1}{2}\vec{c}$)

$$\begin{cases} \vec{a} = \frac{2\pi}{a}(1,-1,1) \\ \vec{b} = \frac{2\pi}{a}(1,1,-1) \\ \vec{c} = \frac{2\pi}{a}(-1,1,1) \end{cases}$$

Then $k_i = \frac{1}{2}\frac{2\pi}{a}\sqrt{1^2+1^2+1^2} = \frac{\sqrt{3}\,\pi}{a}$.

For fcc, the # of atoms is

$\frac{4}{a^3}$ & the # of $\bar{e}$'s is (per unit volume)

$$\frac{4}{a^3}\underbrace{\frac{n}{n_a}}_{\bar{e}\text{-to-atom ratio}}$$

Then

$$\left(3\pi^2 \frac{4}{a^3}\frac{n}{n_a}\right)^{1/3} = \frac{\sqrt{3}\,\pi}{a}, \text{ or}$$

$$\frac{n}{n_a} = \frac{1}{12\cancel{\pi^2}}\, 3^{3/2}\, \pi^{\cancel{3}} = \frac{\sqrt{3}\,\pi}{4} = \underline{\underline{1.36}}.$$

c) Zn divalent

Cu monovalent

$$n = \frac{4}{a^3}\left\{(1-x)\times 1 + x\times 2\right\} = \frac{4}{a^3}(1+x).$$

$\bar{e}$ concentr'n  Zn fraction

$k_F = k_i$ as in (b):

$$(3\pi^2 n)^{1/3} = \frac{\sqrt{3}\,\pi}{a}, \text{ or}$$

$$(12\pi^2)^{1/3}(1+x)^{1/3} = \sqrt{3}\,\pi,$$

$$\left(\frac{12}{\pi}\right)^{1/3}(1+x)^{1/3} = \sqrt{3} \Rightarrow 1+x = \frac{\pi}{12}3\sqrt{3} =$$

$$= \frac{\pi\sqrt{3}}{4} \Rightarrow x = \frac{\sqrt{3}\,\pi}{4} - 1 = \underline{\underline{0.36}},$$

consistent with (b).

8. Kittel Pr. 6

The central equation is

$$(\lambda_k - \epsilon) C_k + \sum_G U_G C_{k-G} = 0 .$$

We recall that $\lambda_k = \frac{\hbar^2 k^2}{2m}$

$$U(x) = \sum_{\vec{G}} U_{\vec{G}} e^{i\vec{G}\vec{x}}$$

$$\Downarrow$$

$$U_{\vec{G}} = \frac{1}{a^2} \int_{cell} U(x) e^{-i\vec{G}\cdot\vec{x}} dx\, dy$$

$$\vec{x} = \{x, y\} \equiv (x_1, x_2)$$

$$\vec{G} = \left(\frac{2\pi}{a}, \frac{2\pi}{a}\right)$$

Then $$U_{\vec{G}} = -\frac{4U}{a^2} \prod_{i=1}^{2} \int_0^a dx_i \cos\left(\frac{2\pi x_i}{a}\right) e^{-i\frac{2\pi x_i}{a}} =$$

$$= -\frac{4U}{a^2} \prod_{i=1}^{2} \int_0^a dx_i \frac{e^{i\frac{2\pi x_i}{a}} + e^{-i\frac{2\pi x_i}{a}}}{2} e^{-i\frac{2\pi x_i}{a}} =$$

$$= -\frac{U}{a^2} \times a \times a = \underline{\underline{-U}} .$$

using $e^{2\pi i n} = 1$

Then the central eq'n is:

$$\begin{bmatrix} \lambda_k - \epsilon & -u \\ -u & \lambda_{k-G} - \epsilon \end{bmatrix} \begin{bmatrix} C_k \\ C_{k-G} \end{bmatrix} = 0$$

$$\det [\ ] = 0 \Rightarrow (\lambda_k - \epsilon)(\lambda_{k-G} - \epsilon) - u^2 = 0$$

$$\vec{k} = \left(\frac{\pi}{a}, \frac{\pi}{a}\right) \qquad \Rightarrow \vec{k} - \vec{G} = \left(-\frac{\pi}{a}, -\frac{\pi}{a}\right)$$

$$\vec{G} = \left(\frac{2\pi}{a}, \frac{2\pi}{a}\right)$$

$$\Downarrow$$

$$\lambda_k = \lambda_{k-G} \equiv \lambda$$

Hence $(\lambda - \epsilon)^2 = u^2$, or

$$\epsilon_\pm = \lambda \pm u .$$

Gap $= \epsilon_+ - \epsilon_- = \underline{\underline{2u}}$.

9. Photonic crystals are devices in which a distribution of refractive indices is chosen such that incoming light waves become standing waves due to refraction & reflection. This makes them analogous to semiconductor devices, except light waves are used instead of $\bar{e}$ waves. Some of the challenges are:

a) identifying suitable materials (with necessary optical properties) & building structures out of them

b) the devices must be able to handle incoming waves coming in all directions

Potential uses: lasers, fiber optics

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