1. Ch. 4, Q. 5

With thermal conduction, $\bar{E}$'s at one end of the sample have more energy than $\bar{E}$'s at the other end. More energetic $\bar{E}$'s diffuse down the $T$ gradient, carrying a net energy flux. On average, there is no particle or charge buildup which would be quite unfavorable energetically.

2. Ch. 4, Q. 8

The Hall constant is defined as

$$R_H = \frac{E_H}{J_x B},$$

where $E_H$ is the Hall field, $B$ is the external magnetic field, and $J_x$ is the current density. Since $J_x \sim N$, ($\bar{E}$ cons'n)

$$R_H \sim \frac{1}{N}.$$
3. Ch. 4, Q. 9

Positive charge carriers mean that \( j_x^+ = N e v_x^+ \)

\[
\nabla \downarrow
\]

\[
\rho_H = \frac{E_H}{j_x B} \sim + \frac{1}{e} > 0
\]
Here we must distinguish between core (localized) and valence (delocalized) \( \bar{\varepsilon} \)'s. The combination of the core \( \bar{\varepsilon} \)'s & the ions produces weak periodic pseudo potential. In other words, core \( \bar{\varepsilon} \)'s "screen" the ions. Bloch's theorem states that the periodic nature of the resulting potential leads to valence \( \bar{\varepsilon} \)'s with propagating (delocalized) wavefunctions. Thus there is no paradox - rather there are 2 types of \( \bar{\varepsilon} \)'s.
5. \( \text{Ch.5, Q.2} \)

For a truly free \( \psi \),

\[
\begin{align*}
\psi_k^{(0)} &= \frac{1}{\sqrt{L}} e^{ikx}, \\
E_k^{(0)} &= \frac{k^2k^2}{2m_0}.
\end{align*}
\]

This leads to a single dispersion curve:

"Cutting & pasting" is not justifiable here because there is no periodicity of the lattice:

\[
E_k = E_{k+G}, \quad G = \frac{2\pi n}{a}, \quad n = 0, \pm 1, \pm 2, \ldots
\]

Thus the difference between empty lattice & free space is that we impose symmetry properties in \( k \)-space (even though there is no potential) in the former case. These symmetries follow from the translational symmetry of the real lattice.
6. Ch. 4, Pr. 10

\[ V_c = \frac{19c}{25c} = 2.8 \text{ B GHz} \quad \text{for} \quad m^* = m_0, \]

\[ \text{in kg} \]

Then \[ V_c = 24 \text{ GHz} \quad \text{gives} \]

\[ b = \frac{24}{2.8} \approx 8.6 \text{ kG} \]

7. Ch. 5, Pr. 14

a) Free \( \bar{e} \) model:

\[ n = \frac{2}{7} \left( \frac{1}{(2\pi)^3} \right) \frac{1}{3} \frac{4}{3} \alpha k_F^3 = \frac{1}{35c^2} k_F^3, \quad \text{or} \]

Spin \[ k_F = (35c^2 n)^{\frac{1}{3}} \]

b) Fermi sphere will touch the face of the 1st BZ when \[ k_F = k_i, \quad \text{where} \]

\[ k_i = \frac{4}{7} \frac{a_0}{2a} \quad \text{for fcc, and} \]

\[ \text{(or} \frac{4}{7} \frac{b_0}{2b}, \frac{4}{7} \frac{c_0}{2c}) \]

\[ \left\{ \begin{array}{c}
\bar{a} = \frac{25c}{a} (1, -1, 1) \\
\bar{b} = \frac{25c}{b} (1, 1, -1) \\
\bar{c} = \frac{25c}{c} (-1, 1, 1)
\end{array} \right. \]
Then \( k_i = \frac{1}{2} \frac{25\pi}{a} \sqrt{1^2 + 1^2 + 1^2} = \frac{\sqrt{3} \pi}{a} \).

Forfcc, the # of atoms is 
\[ \frac{\nu}{a^3} \]
and the # of \( \tilde{e} \)'s is (per unit volume)
\[ \frac{\nu}{a^3} \frac{n}{N_a} \]
\[ \tilde{e}-to-atom\ ratio \]

Then
\[ (\frac{3\pi}{2} - \frac{\nu}{a^3} \frac{n}{N_a})^{1/3} = \frac{\sqrt{3} \pi}{a}, \quad \text{or} \]
\[ \frac{n}{N_a} = \frac{1}{12 \pi} \frac{3^{1/2}}{\pi^{3/2}} \frac{3}{4} = \frac{\sqrt{3} \pi}{\nu} = 1.36. \]

\( c) \quad \text{Zn}\quad \text{trivalent} \]
\( \text{Cu}\quad \text{monovalent} \]
\[ n = \frac{\nu}{a^3} \left[ (1-x) \times 1 + x \times 2 \right] = \frac{\nu}{a^3} (1+x) \]
\[ \tilde{e} \text{ concntr'n} \]
\[ k_F = k_i \quad \text{as in (b)}: \]
\[ (3\pi/2 \nu \times a)^{1/3} = \frac{\sqrt{3} \pi}{a}, \quad \text{or} \]
\[ (12 \pi^2)^{1/3} (1+x)^{1/3} = \sqrt{3} \pi, \]
\[ (\frac{12 \pi}{3})^{1/3} (1+x)^{1/3} = \sqrt{3} \rightarrow 1+x = \frac{\pi}{12} \cdot 3 \cdot \sqrt{3} = \]
\[ = \frac{\sqrt{3}}{\nu} \rightarrow x = \frac{13 \pi}{\nu} - 1 \approx 0.36, \]
consistent with (b).
The central equation is

\[(\lambda_k - \epsilon) C_k + \sum_G U_G C_{k-G} = 0.\]

We recall that \[\lambda_k = \frac{\hbar^2 k^2}{2m}\]

\[U(x) = \sum_G U_G e^{-iG \cdot x}\]

\[\downarrow\]

\[U_G = \frac{1}{a^2} \iint_{cell} U(x) e^{-iG \cdot x} \, dx \, dy\]

\[\mathbf{x} = \{x, y\} \equiv (x_1, x_2)\]

\[\mathbf{G} = \left( \frac{2\pi}{a}, \frac{2\pi}{a} \right)\]

Then

\[U_G = -\frac{4\pi}{a^2} \prod_{i=1}^{2} \int_{0}^{a} dx_i \cos \left( \frac{2\pi x_i}{a} \right) e^{-i \frac{2\pi x_i}{a}} =\]

\[= -\frac{4\pi}{a^2} \prod_{i=1}^{2} \int_{0}^{a} dx_i \frac{e^{i \frac{2\pi x_i}{a}} + e^{-i \frac{2\pi x_i}{a}}}{2} e^{-i \frac{2\pi x_i}{a}} =\]

\[= -\frac{u}{a^2} \times a \times a = -u\]

\[\text{using } e^{2\pi i n} = 1\]
Then the central eq'n is:

\[
\begin{bmatrix}
\lambda_k - \epsilon & -\mathbf{u} \\
-\mathbf{u} & \lambda_{k-G} - \epsilon
\end{bmatrix}
\begin{bmatrix}
\mathbf{c}_k \\
\mathbf{c}_{k-G}
\end{bmatrix} = 0
\]

\[
\det \begin{bmatrix}
\lambda_k - \epsilon & -\mathbf{u} \\
-\mathbf{u} & \lambda_{k-G} - \epsilon
\end{bmatrix} = 0 \Rightarrow (\lambda_k - \epsilon)(\lambda_{k-G} - \epsilon) - \mathbf{u}^2 = 0
\]

\[
\mathbf{k} = \left( \frac{\pi}{a}, \frac{\pi}{a} \right) \Rightarrow \mathbf{k} - \mathbf{G} = \left( -\frac{\pi}{a}, -\frac{\pi}{a} \right)
\]

\[
\mathbf{G} = \left( \frac{2\pi}{a}, \frac{2\pi}{a} \right)
\]

\[
\lambda_k = \lambda_{k-G} \equiv \lambda
\]

Hence \((\lambda - \epsilon)^2 = \mathbf{u}^2\), or

\[
\epsilon_{\pm} = \lambda \pm \mathbf{u}
\]

\[
\text{Gap} = \epsilon_+ - \epsilon_- = 2\mathbf{u}
\]
Photonic crystals are devices in which a distribution of refractive indices is chosen such that incoming light waves become standing waves due to refraction & reflection. This makes them analogous to semiconductor devices, except light waves are used instead of e-waves. Some of the challenges are:

a) identifying & building structures suitable materials out of them (with necessary optical properties)

b) the devices must be able to handle incoming waves coming in all directions

Potential uses: lasers, fiber optics