1. Ch. 4, Q. 2

In a plasma, the particles are charged, whereas in a gas, they are usually neutral. One can imagine conduction electrons as a dense plasma: \( N \approx 10^{29} \) electrons/m\(^3\), compared with \( N \approx 10^{25} \) molecules/m\(^3\) for ordinary gas.

2. Ch. 4, Q. 6

\[ \rho = \frac{1}{6} = \frac{m}{Ne^2 \tau} \]

\[ \tau = \frac{e}{v} \quad \text{and} \quad \frac{e}{6N} = 1 \]

\[ e \sim \frac{1}{6n} \]

Random thermal motion \( \Rightarrow \)

\( \Rightarrow \) equipartition theorem:

\[ \frac{m}{2} \langle v^2 \rangle = \frac{m(u^2)}{2} \langle x^2 \rangle \sim k_B T \]

Then

\[ \ell \sim \frac{1}{6} \sim \frac{1}{\pi \langle x^2 \rangle} \sim \frac{1}{T} \]

Cross-section due to thermal fluctuations.
Also, \( v \sim T^{1/2} \).

Thus \( p \sim \frac{1}{T} \sim T^{-3/2} \)

\[ \text{(3) Or Pr. 2)} \]

Mean Free Path \( l = \tau \nu \), such that \( \frac{dx}{l} \) is the mean free path the probability to have a collision in going from \( x \) to \( x + dx \).

\[ N_i \, dx \sim \text{total # of impurities} \]

\[ A = 1 \] (unit area)

Consider a particle which travels distance \( dx \) with \( N_i \) impurities per unit volume. Each impurity has a cross-section (effective collision area) \( \sigma_i \). Then the total area covered by the scatterers is \( \sigma_i \, N_i \, dx \) (A=1).

Thus \[ \frac{\sigma_i \, N_i \, dx}{l} = \frac{dx}{l} \Rightarrow l = \frac{1}{\sigma_i \, N_i} \] prob. to have a collision.
4. Pr. 6

From Table 4.1 in Omar we have:

<table>
<thead>
<tr>
<th>Element</th>
<th>( E_F ) (eV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cu</td>
<td>7.0</td>
</tr>
<tr>
<td>Na</td>
<td>3.1</td>
</tr>
<tr>
<td>Ag</td>
<td>5.5</td>
</tr>
</tbody>
</table>

Since \( T_F = \frac{E_F}{k_B} \) \& \( T = 300 \text{ K} \), we obtain:

<table>
<thead>
<tr>
<th>Element</th>
<th>( T_F, \text{K} )</th>
<th>( T/T_F )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cu</td>
<td>( 8 \times 10^4 )</td>
<td>( 3.8 \times 10^{-3} )</td>
</tr>
<tr>
<td>Na</td>
<td>( 3.6 \times 10^4 )</td>
<td>( 8.3 \times 10^{-3} )</td>
</tr>
<tr>
<td>Ag</td>
<td>( 6.4 \times 10^4 )</td>
<td>( 4.6 \times 10^{-3} )</td>
</tr>
</tbody>
</table>
(5) \(8\). Pr. 7

Fraction of \(e\)'s excited above Fermi level at \(T = 300\) K

\[
\frac{k_B T}{E_F} = \frac{T}{T_F},
\]

So we can use results from Pr. 6

\[
\text{Cu} \Rightarrow f_{\text{Cu}} = 3.8 \times 10^{-3}
\]

\[
\text{Na} \Rightarrow f_{\text{Na}} = 8.3 \times 10^{-3}
\]

(6) Current density \(j = 10 \frac{\text{Amp}}{\text{mm}^2} = 10 \frac{\text{C/s}}{10^{-6} \text{m}^2}\)

\[
\frac{\nu_d}{\nu_F} \text{ for Cu wire?}
\]

From Table 4.1 in "\(\)"

\[
\text{Cu} \left\{ \begin{align*}
\nu_F &= 1.6 \times 10^6 \text{ m/s} \\
N &= 8.45 \times 10^{28} \text{ e/m}^3
\end{align*} \right.
\]

\[
\nu_d = \frac{j}{Ne} = \frac{10 \text{ C/s}}{10^{-6} \text{m}^2} = \frac{1000 \text{ m/s}}{8.45 \times 10^{28} \text{ e/m}^3} (1.6 \times 10^{-19} \text{ C/e})
\]

\[
\approx 7.4 \times 10^{-11} \text{ m/s}
\]

Thus \(\frac{\nu_d}{\nu_F} = \frac{7.4 \times 10^{-11} \text{ m/s}}{1.6 \times 10^6 \text{ m/s}} \approx 4.6 \times 10^{-10}\)
7. Steady-state:

\[ m \frac{d\mathbf{v}_d}{dt} = -\mathbf{E} + m \frac{\gamma_d}{\tau} = 0, \quad \text{or} \]

\[ \mathbf{v}_d = -\frac{\mathbf{E}}{m} \tau \]

Current density

\[ j = -ne\mathbf{v}_d = -\frac{ne^2 \tau}{m} \quad \mathbf{E} = \delta \mathbf{E} \]

Ohm’s law

Thus \( \delta = \frac{ne^2 \tau}{m} \), where

\( \tau \) is the effective collision time

\& \( m \) is the effective \( e \) mass.

8. Liquid \( \text{He}^3 \):

\[ \epsilon_F = \frac{\hbar^2}{2m} \left( 3 \pi^2 \bar{n} \right)^{2/3} \]

\[ \rho = 0.081 \text{ g/cm}^3 \]

\[ \frac{\# \text{ moles}}{\text{cm}^3} = \frac{8.1 \times 10^{-2} \text{ g/cm}^3}{3 \text{ g/mole}} = 2.7 \times 10^{-2} \frac{\text{ moles}}{\text{cm}^3}. \]

Concentration of \( \text{He} \) atoms

\[ N_H = 2.7 \times 10^{-2} \frac{\text{ moles}}{\text{cm}^3} \times 6 \times 10^{23} \frac{\text{ atoms}}{\text{ mole}} = \]

\[ = 1.6 \times 10^{22} \text{ atoms/cm}^3. \]
\[ m_H = 3 m_p = 3 \times 1.6 \times 10^{-24} g = 4.8 \times 10^{-24} g. \]

Thus
\[ E_F = \frac{\hbar^2}{2m_H} (\frac{3\pi}{2} h_H)^{2/3} \approx \]
\[ \approx 6 \times 10^{-16} \text{ erg}. \]

Finally,
\[ T_F = \frac{E_F}{k_B} = \frac{6 \times 10^{-16} \text{ erg}}{1.4 \times 10^{-16} \text{ erg}/K} = 4.3 K. \]
The article describes how to create a low-T gas of fermionic atoms. Whereas bosons fall into the ground level at low T, fermions cannot share quantum states due to the Pauli exclusion principle. Thus cooling through head-on "s-wave" collisions is not possible for fermions. To circumvent this difficulty, Jih's group used atoms in two distinct spin states; Hulet's group used mixtures of isotopes. In both cases, quantum degeneracy was achieved via collisions, using evaporative cooling.