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HW #4 solutions

1. O. Ch. 4, Q. 2

In a plasma the particles are charged, whereas in a gas they are usually neutral. One can imagine conduction electrons as a dense plasma: $N \sim 10^{29}$ electrons/m^3, compared with $N \sim 10^{25}$ molecules/m^3 for ordinary gas.

2. O. Ch. 4, Q. 6

$$\rho = \frac{1}{\sigma} = \frac{m}{Ne^2\tau}$$

$\tau = l/v$, and

$$l \underbrace{\sigma}_{\text{cross-section}} \underbrace{n}_{\bar{e}\text{ concentration}} = 1 \Downarrow l \sim \frac{1}{\sigma n}$$

Random thermal motion $\Rightarrow$

$\Rightarrow$ equipartition theorem:

$$\frac{m}{2}\langle v^2\rangle \sim \frac{m\omega^2}{2}\langle x^2\rangle \sim k_B T$$

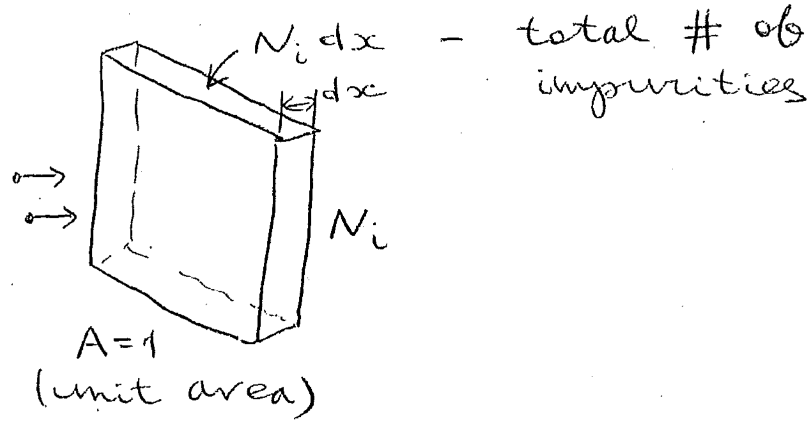
Then
$$l \sim \frac{1}{\sigma} \sim \frac{1}{\underbrace{\pi\langle x^2\rangle}_{\text{cross-section due to thermal fluctuations}}} \sim \frac{1}{T}$$

Also, $\vartheta \sim T^{1/2}$.

Thus $\rho \sim \frac{1}{\tau} \sim T^{3/2}$

(3.) O Pr. 2

MFP $\ell = \tau\vartheta$, such that $\frac{dx}{\ell}$ is
mean free path
the probability to have a collision in going from x to $x+dx$.



Consider a particle which travels distance dx through material with N_i impurities per unit volume. Each impurity has a cross-section (effective collision area) σ_i. Then the total area covered by the scatterers is $\sigma_i N_i dx$ $(A=1)$.

Thus
$$\underbrace{\sigma_i N_i dx}_{\text{prob. to have a collision}} = \frac{dx}{\ell} \Rightarrow \ell = \frac{1}{\sigma_i N_i}$$

(4.) O. Pr. 6

From Table 4.1 in Omar we have:

Element	E_F (eV)
Cu	7.0
Na	3.1
Ag	5.5

Since $T_F = \frac{E_F}{k_B}$ & $T = 300\,K$, we obtain:

Element	T_F, K	T/T_F
Cu	8×10^4	3.8×10^{-3}
Na	3.6×10^4	8.3×10^{-3}
Ag	6.4×10^4	4.6×10^{-3}

5. ϑ. Pr. 7

Fraction of $\bar{e}$'s excited ~~above~~ above Fermi level at $T = 300\,K$ $\simeq \frac{k_B T}{E_F} = \frac{T}{T_F}$,

good enough for an estimate

so we can use results from Pr. 6

$$Cu \Rightarrow f_{Cu} = 3.8 \times 10^{-3}$$

$$Na \Rightarrow f_{Na} = 8.3 \times 10^{-3}$$

6. Current density $j = 10 \frac{Amp}{mm^2} = 10 \frac{C/s}{10^{-6}\,m^2}$

$\frac{\vartheta_d}{\vartheta_F}$ for Cu wire - ?

From Table 4.1 in Omar:

$$Cu \begin{cases} \vartheta_F = 1.6 \times 10^6\ m/s \\ N = 8.45 \times 10^{28}\ e/m^3 \end{cases}$$

$$j = Ne\vartheta_d \Rightarrow \vartheta_d = \frac{j}{Ne} =$$

$$= \frac{10\ C/s\ /10^{-6}\,m^2}{(8.4 \times 10^{28}\ e/m^3)(1.6 \times 10^{-19}\ C/e)} \simeq$$

$$\simeq 7.4 \times 10^{-4}\ m/s$$

Thus $\frac{\vartheta_d}{\vartheta_F} = \frac{7.4 \times 10^{-4}\ m/s}{1.6 \times 10^6\ m/s} \simeq \underline{\underline{4.6 \times 10^{-10}}}$

(7.) Steady-state:

$$m\frac{dv_d}{dt} = -eE - m\frac{v_d}{\tau} = 0, \text{ or}$$

$$v_d = -\frac{eE}{m}\tau.$$

Current density

$$j = -nev_d = \frac{ne^2\tau}{m}E = \sigma E$$

Ohm's law

Thus $\sigma = \dfrac{ne^2\tau}{m}$, where τ is the effective collision time & m is the effective $\bar{e}$ mass.

(8.) Liquid He^3.

$$\epsilon_F = \frac{\hbar^2}{2m}(3\pi^2 n)^{2/3}$$

$$\rho = 0.081\ g/cm^3$$

$$\frac{\#\ moles}{cm^3} = \frac{8.1\times10^{-2}\ g/cm^3}{3\ g/mole} = 2.7\times10^{-2}\ \frac{moles}{cm^3}.$$

Concentration of He atoms

$$n_H = 2.7\times10^{-2}\ \frac{moles}{cm^3} \times 6\times10^{23}\ \frac{atoms}{mole} =$$

$$= 1.6\times10^{22}\ atoms/cm^3.$$

proton mass

$$m_H = 3\,\underset{\swarrow}{m_p} = 3 \times 1.6 \times 10^{-24}\,g =$$

$$= 4.8 \times 10^{-24}\,g.$$

Thus $$\epsilon_F = \frac{\hbar^2}{2m_H}\left(3\pi^2 n_H\right)^{2/3} \simeq$$

$$\simeq 6 \times 10^{-16}\ \underline{\underline{erg}}.$$

Finally,

$$T_F = \frac{\epsilon_F}{k_B} = \frac{6\times10^{-16}\,erg}{1.4\times10^{-16}\,erg/K} = \underline{\underline{4.3\ K}}$$

(9.) The article describes how to create a low-T gas of fermionic atoms. Whereas bosons fall into the ground level at low T, fermions cannot share quantum states due to the Pauli exclusion principle. Thus cooling through head-on "s-wave" collisions is not possible for fermions. To circumvent this difficulty, Jin's group used atoms in two distinct spin states; Hulet's group used mixtures of isotopes. In both cases quantum degeneracy was achieved via collisions, using evaporative cooling.