Physics 313
Midterm Exam Solutions
October 21, 2014

The exam is 80 minutes in length.

There are five problems. All are worth the same amount of points.

You may refer to a single handwritten note sheet on $8\frac{1}{2}'' \times 11''$ paper (double-sided) that you brought with you.

Good Luck!

Problem 1

In atomic hydrogen, the electron with mass approximately 0.5 MeV/c^2 is confined to a distance of about 1 Å (1×10^{-10} m), the radius of the hydrogen atom. The resulting uncertainty in its momentum is about 1×10^{-3} MeV/c.

a) What is the uncertainty in the velocity of the electron?

$$\Delta p \;=\; m\Delta v \qquad \Rightarrow \qquad \Delta v \;=\; \frac{\Delta p}{m} \;=\; \frac{10^{-3}\ \text{MeV/c}}{0.5\ \text{MeV/c}^2} \;=\; 2 \times 10^{-3} c$$

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b) Is the electron relativistic?

No. $\quad \dfrac{v}{c} = 0.002 \quad \Rightarrow \quad \gamma = \dfrac{1}{\sqrt{1-(2\times 10^{-3})^2}} = \dfrac{1}{\sqrt{1-4\times 10^{-6}}} = 1.000002$

The down quark has a mass of approximately 5 MeV/c^2. It is a constituent of the proton and is confined to a distance of about 1 fm (1×10^{-15} m), the radius of the proton.

c) What is the uncertainty in the momentum of the down quark inside the proton?

Since the position uncertainty is 10^5 smaller than for the electron in the hydrogen atom, the momentum uncertainty will be 10^5 larger $\quad \Rightarrow \quad \Delta p \;=\; 100$ MeV/c

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d) Is the down quark inside the proton relativistic? [Hint: what is its γ factor?]

$$\gamma v \;=\; \frac{p}{m} \;=\; \frac{100\ \text{MeV/c}}{5\ \text{Mev/c}^2} \;=\; 20c \qquad \Rightarrow \qquad \gamma = 20$$

Problem 2

In class, I mentioned five problems with classical physics that physicists were confronted with around the beginning of the 20th century.

a) List three of these and briefly describe why it is inconsistent with a classical physics interpretation.

- Spectrum of black body radiation. Classical physics gives a spectral intensity that monotonically increases with frequency of the radiation.
- Photoelectric effect. In classical physics, the energy of the ejected electrons should depend only on the intensity of the incident light and be independent of the frequency. Also, there shouldn't be a frequency dependent threshold.
- Discreet atomic spectral lines. In classical physics, excited atoms should emit a continuous spectrum of electromagnetic radiation from acceleration of the electrons. Since the frequency of the acceleration can be anything the spectrum should be continuous.
- Stability of atoms. If we model the electrons in an atom as a tiny classical planetary system, the electrons would radiate because they are charged and are undergoing centripetal acceleration. They would spiral into the nucleus in about 1 ns.
- Specific heats of diatomic gases. By the classical equipartition theorem, each degree of freedom has $(1/2)k_BT$ of energy. The molar specific heat of a diatomic gas should then be $(7/2)R$ independent of temperature.

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b) Choose one of these and describe in more detail the initial attempts to resolve the problem and the clues that this gave concerning the nature of the emerging theory of quantum mechanics.

- Spectrum of black body radiation. In order to explain the blackbody spectra, Planck had too assume that the walls f the blackbody cavity could only radiate energy in quanta of hf.
- Photoelectric effect. Einstein explained the photoelectric effect by postulating that light (electromagnetic radiation) was quantized in photons each of which carried and energy of hf.
- Discreet atomic spectral lines. Although we haven't discussed this in class, Bohr tried to explain this by using Einstein's postulate that light energy came in quanta of hf combined with the angular momentum of the atomic electrons being quantized in integral amounts of $\hbar$.
- Stability of atoms. Bohr's mddl gave a ground state of the atom in which the electron was at a non zero radius. It couldn't spiral into the nucleus because

the minimum value of angular momentum that it could have is $\hbar$. Bohr's was a valiant early attempt but we now know that his model was inconsistent and wrong. We'll see the correct theory of the hydrogen atom n the coming few lectures.

– Specific heats of diatomic gases. The vibrational and rotational modes of the diatomic atom seem to be quantized so that there is a minimum energy of rotation and vibration. If the average thermal energy $(3/2)k_BT$ is much smaller than this, then the rotational and vibrational degrees of freedom are "frozen out."

Problem 3

Physicists working at the Large Hadron Collider at CERN have found an event in which two high energy photons were produced. One of the photons has an energy of 50 GeV and the other an energy of 530 GeV. The angle between the directions of the two photons is 45^o.

a) Assuming that these two photons came from the decay of a mother particle, what is the mass of this mother particle?

Work in the system of unit in which $c = 1$.

$$M^2 = (P_1^\mu + (P_2^\mu)(P_{\mu,1} + P_{\mu,2}) = 2(E_1 E_2 - \vec{p_1} \cdot \vec{p_2}) = 2E_1 E_2 (1 - \cos\theta)$$

$$= (2)(50 \text{ GeV})(530 \text{ GeV}) \left(1 - \frac{1}{\sqrt{2}}\right) = 125 \text{ GeV}$$

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b) What was the velocity of the mother particle in the lab frame before it decayed?

$$\gamma = \frac{E}{M} = \frac{E_1 + E_2}{M} = \frac{580 \text{ GeV}}{125\text{GeV}} = 4.64$$

$$\frac{v}{c} = \sqrt{1 - \frac{1}{\gamma^2}} = 0.9765$$

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c) For extra credit, what is the name of this particle?

This is the famous Higgs Boson. That is the most important (perhaps only) discovery made at the LHC to date.

Problem 4

In one of the homework problems, you saw that the wave function of the bound state of the delta potential well is given by:

$$\psi(x) \;=\; \sqrt{\alpha}\, e^{-\alpha|x|}$$

That is, the wave function is symmetric about $x = 0$ with

$$\psi(x) \;=\; \begin{cases} \sqrt{\alpha}\, e^{-\alpha x} & \text{for } x > 0 \\ \sqrt{\alpha}\, e^{\alpha x} & \text{for } x < 0 \end{cases}$$

For this wave function, determine the following.

a) The expectation value of x, $\langle x \rangle$.

By symmetry, $\langle x \rangle \;=\; 0$

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b) The expectation value of x^2, $\langle x^2 \rangle$.

$$\langle x^2 \rangle \;=\; \int_{-\infty}^{\infty} \psi^*(x) x^2 \psi(x)\, dx \;=\; 2\int_0^{\infty} \sqrt{\alpha}\, e^{-\alpha x} x^2 \sqrt{\alpha}\, e^{-\alpha x}\, dx \;=\; 2\alpha \int_0^{\infty} e^{-\alpha x} x^2\, e^{-\alpha x}\, dx$$

$$= \; 2\alpha \int_0^{\infty} x^2\, e^{-2\alpha x}\, dx \;=\; 2\alpha \left(\frac{2}{(2\alpha)^3} \right) \;=\; \frac{1}{2\alpha^2}$$

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c) The rms of x, σ_x.

$$\sigma_x \;=\; \sqrt{\langle x^2 \rangle \;-\; \langle x \rangle^2} \;=\; \sqrt{\frac{1}{2\alpha^2}} \;=\; \frac{1}{\sqrt{2}\alpha}$$

For all parts, show your calculation or your reasoning.

Problem 5

Bob is standing at rest on a planet and watches Anne in her hyper drive spaceship of length 2 m pass by at a speed of $0.9c$. Anne is sitting at the rear of her ship and when she passes Bob the two of them synchronize their clocks to $t = 0$. Mary standing in the front of Anne's ship jumps into the air. According to clocks in Anne's frame, Mary jumps at the same time that Anne passes Bob, $t = 0$.

a) According to clocks in Bob's frame, what time relative to $t = 0$ did Mary jump?

Let Anne's be the prime frame and Bob's be the unprimed frame.

$$\Delta x' = 2 \text{ m} \qquad \Delta t' = 0$$

$$\Delta t = \gamma(\Delta t' + \frac{v}{c^2}\Delta x') = \gamma \frac{v}{c^2}\Delta x'$$

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}} = \frac{1}{\sqrt{1 - (0.9)^2}} = 2.29$$

$$\Delta t = (2.29)\left(\frac{0.9}{c}\right)(2 \text{ m}) = (2.29)\left(\frac{0.9}{3 \times 10^8 \text{ m/s}}\right)(2 \text{ m}) = 1.37 \times 10^{-8} \text{ s} = 13.7 \text{ ns}$$

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b) According to Mary, she is in the air for 0.5 s. How far from where she jumped does an observer in Bob's frame see her land?

By time dilation, an observer in Bob's frame sees Mary's clock running slow by a factor of γ. So the observer in Bob's frame sees that Mary is in the air for $\Delta t = (2.29)(0.5 \text{ s}) = 1.15$ s. Since Mary, in Anne's ship, is moving with a speed of $0.9c$ with respect to Bob's frame, she will land a distance

$$d = v\Delta t = (0.9)(3 \times 10^8 \text{ m/s})(1.15 \text{ s}) = 3.1 \times 10^8 \text{ m}$$

from where she jumped.
