Solution 9

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9.61 With one conduction electron per atom, the number of conduction electrons per unit volume equals the number of atoms per unit volume. $\frac{8.9\times10^3\,\text{kg/m}^3}{63.5\text{u}\times1.66\times10^{-27}\,\text{kg/u}} = 8.44\times10^{28}\,\text{m}^{-3}$. $E_F = \frac{\pi^2\hbar^2}{9.11\times10^{-31}\text{kg}}\left(\frac{3}{(2)\pi\sqrt{2}}8.44\times10^{28}\,\text{m}^{-3}\right)^{2/3}$ $= 1.13\times10^{-18}\text{J} = \mathbf{7.0eV}$

9.67 Again assuming T = 0, so that $\mathcal{N}(E)_{FD} = 1$ up to E_F and is thereafter zero, $U_{total} = \int E\,\mathcal{N}(E)\,D(E)\,dE$ $= \int_0^{E_F} E\,(1)\frac{(2s+1)m^{3/2}V}{\pi^2\hbar^3\sqrt{2}}E^{1/2}dE = \frac{(2s+1)m^{3/2}V}{\pi^2\hbar^3\sqrt{2}}\frac{2}{5}E_F^{5/2}$ Now we need only eliminate E_F. Inserting (9-42), and noting that $2s+1$ is two for spin$-\frac{1}{2}$, $U_{total} = \frac{\sqrt{2}m^{3/2}V}{\pi^2\hbar^3}\frac{2}{5}\left(\frac{\pi^2\hbar^2}{m}\left(\frac{3}{2\pi\sqrt{2}}\frac{N}{V}\right)^{2/3}\right)^{5/2} = \frac{3}{10}\left(\frac{3\pi^2\hbar^3}{m^{3/2}V}\right)^{2/3}N^{5/3}$.

9.68 $\bar{E} = \frac{U_{total}}{N} = \frac{3}{10}\left(\frac{3\pi^2\hbar^3}{m^{3/2}V}\right)^{2/3}N^{2/3} = \frac{3\pi^2\hbar^2}{10m}\left(\frac{3}{\pi}\frac{N}{V}\right)^{2/3} = \frac{3\pi^2\hbar^2}{10m}\left(\frac{3\cdot2^{3/2}}{(2\frac{1}{2}+1)2^{1/2}\pi}\frac{N}{V}\right)^{2/3}$

$= \frac{3\pi^2\hbar^2}{5m}\left(\frac{3}{(2\frac{1}{2}+1)2^{1/2}\pi}\frac{N}{V}\right)^{2/3}$. This is 3/5 times E_F, as given in equation (9-42).

10.26 **I − II gives a bump in the middle atom** of double-height. I + II gives double-height bumps in the outside atoms, so **III ± (I + II)/2 gives bumps in either of the outside atoms**, also of double-height.

(b) **I, III, II**. In this order they resemble the ground, first-excited, and next-excited states in a single well. Also, as the atoms draw close, penetration of the classical forbidden region would be more pronounced. Wave I would be entirely above the horizontal axis, essentially a single antinode with a half-wavelength stretching from one end of the unit to the other. Wave II would still have a node in the middle. Giving a shorter wavelength, and wave III would still have two nodes.

10.27 The proton repulsion energy is $\frac{ke^2}{r} = \frac{(9\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2)(1.6\times10^{-19}\text{C})^2}{1.1\times10^{-10}\text{m}} = 2.09\times10^{-18}\text{J} = 13.1\text{eV}$. To give a total energy of −16.3eV, the electron's (kinetic and electrostatic potential) energy must be **−29.4eV**.

10.30 For HF, 13.6eV is required to remove the electron from hydrogen, and 3.4eV is retrieved as it is seized by the fluorine. 10.2eV is needed, so U must be −10.2eV.

$-10.2\times1.6\times10^{-19}\text{J} = -8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2\,\frac{(1.6\times10^{-19}\text{C})^2}{r} \Rightarrow r_{HF} = \mathbf{0.14nm}$.

For NaF, U must be −(5.1−3.4) = −1.7eV. $-1.7\times1.6\times10^{-19}\text{J} = -8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2\,\frac{(1.6\times10^{-19}\text{C})^2}{r} \Rightarrow r_{NaF}$ = **0.85nm**. The molecule in which the atoms are closer will be the one in which electrons are more likely to be shared between atoms. Thus, it must be that the HF is considered to be covalent, and NaF ionic.