Solution 7

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7.56 (a) The angular parts are constant, and the radial part is a simple decaying exponential. The probability per unit volume is thus maximum at the **origin**, as indicated in Figure 7.15.

(b) The most probable radius satisfies $\frac{d}{dr}P(r) = 0$, where the probability per unit *distance* in the radial direction peaks. $\frac{dP(r)}{dr} = \frac{d}{dr}\left(R^2(r)r^2\right) = \frac{d}{dr}\left(Ae^{-2r/a_0}r^2\right) = A\left(-\frac{2r^2}{a_0} - 2r\right)e^{-2r/a_0} = 0 \Rightarrow r = \mathbf{a_0}$ ($r = 0$ and ∞ being *minima* of this function).

(c) The most probable location is the origin, but the "amount of space" at a *given radius* increases as the surface of a sphere, causing the most probable radius to occur at some distance away from the origin.

7.60 $\bar{U} = \int_0^\infty U(r)P(r)dr$. But $P(r) = R^2r^2 = \left(\frac{1}{(1a_0)^{3/2}}2e^{-r/a_0}\right)^2 r^2 = \frac{4}{a_0^3}r^2e^{-2r/a_0}$. Thus

$$\bar{U} = \int_0^\infty\left(\frac{1}{4\pi\varepsilon_0}\frac{-e^2}{r}\right)\frac{4}{a_0^3}r^2e^{-2r/a_0}dr = -\frac{e^2}{\pi\varepsilon_0 a_0^3}\int_0^\infty re^{-2r/a_0}dr = -\frac{e^2}{\pi\varepsilon_0 a_0^3}\frac{1!}{(2/a_0)^2} = -\frac{1}{4\pi\varepsilon_0}\frac{e^2}{a_0}$$

$$= -(8.99\times10^9\text{N}\cdot\text{m}^2/\text{C}^2)\,\frac{(1.6\times10^{-19}\text{C})^2}{0.0529\times10^{-9}\text{m}} = 4.35\times10^{-18}\text{J} = \mathbf{-27.2eV}.$$

(b) The energy is a well-defined −13.6eV, so the expectation value of the KE must be −13.6eV − (−27.2eV) = **+13.6eV**

8.25 10^{-34}J·s = $(10^{-18}\text{m})p \Rightarrow p \cong 10^{-16}$kg·m/s. Dividing by a mass of about 10^{-30}kg gives 10^{14}m/s. It is true that $\frac{p}{m} = \gamma_u u$ can be arbitrarily high, but γ_u would have to be very high.

(b) $(10^{-16}\text{kg}\cdot\text{m/s})(3\times10^8\text{m/s}) \cong 10^{-8}$J. For the electron, $mc^2 \cong (10^{-30}\text{kg})(10^{17}\text{m}^2/\text{s}^2) \cong 10^{-13}$J. The energy of the mass at the electron's equatorial belt would be orders of magnitude larger than the internal energy of the electron.

8.27 The formula obtained in equation (8.2) applies if instead of replacing μ with $-\frac{e}{2m}$**L** (correct for orbital angular momentum) we replace it with $-\frac{e}{m}$**S** (correct for spin). In essence, wherever an $\frac{e}{m}$ appears we should replace it with a $\frac{2e}{m}$, giving $\frac{eB}{m}$, rather than $\frac{eB}{2m}$, for ω. $\frac{(1.6\times10^{-19}\text{C})(1\text{T})}{(9.11\times10^{-31}\text{kg})} = \mathbf{1.76\times10^{11}Hz}$.

8.30 For a magnetic dipole in a uniform field, $U = -\boldsymbol{\mu}\cdot\mathbf{B}$. Assuming **B** is in the z–direction, $U = -\mu_z B_z$.

But $\boldsymbol{\mu} = -\frac{e}{m}\mathbf{S} \Rightarrow \mu_z = -\frac{e}{m}S_z$. Thus $U = -\left(-\frac{e}{m}S_z\right)B_z$, which in turn is $U = \left(\frac{e}{m}\left(\pm\frac{1}{2}\hbar\right)\right)B_z = \pm\frac{e}{m}\frac{1}{2}\hbar B_z$.

$$\Delta U = \frac{e}{m}\hbar B_z = \frac{1.6\times10^{-19}\text{C}}{9.11\times10^{-31}\text{kg}}(1.055\times10^{-34}\text{J}\cdot\text{s})(1\text{T}) = 1.85\times10^{-23}\text{J} = \mathbf{1.16\times10^{-4}eV}$$

8.35 This is the same as the example, but with a 1 and 2, rather than a 4 and 3.

$$\text{Probability} = \int_0^{L/2}\left[\frac{\sqrt{2}}{L}\left(\sin\frac{1\pi x_1}{L}\sin\frac{2\pi x_2}{L} \pm \sin\frac{2\pi x_1}{L}\sin\frac{1\pi x_2}{L}\right)\right]^2 dx_1\, dx_2$$

$$= \frac{2}{L^2}\int_0^{L/2}\sin^2\frac{1\pi x_1}{L}dx_1 \int_0^{L/2}\sin^2\frac{2\pi x_2}{L}dx_2 + \frac{2}{L^2}\int_0^{L/2}\sin^2\frac{2\pi x_1}{L}dx_1 \int_0^{L/2}\sin^2\frac{1\pi x_2}{L}dx_2$$

$$\pm 2\frac{2}{L^2}\int_0^{L/2}.\sin\frac{1\pi x_1}{L}\sin\frac{2\pi x_1}{L}dx_1 \int_0^{L/2}\sin\frac{2\pi x_2}{L}\sin\frac{1\pi x_2}{L}dx_2\,.$$

The first four integrals are $L/4$, and the later two, using the formulas from the example, are $2L/3\pi$. Thus,

$$\text{Probability} = \frac{2}{L^2}\left(\left(\frac{1}{4}L\right)^2 + \left(\frac{1}{4}L\right)^2 \pm 2\left(\frac{2L}{3\pi}\right)^2\right) = \frac{1}{4} \pm \frac{16}{9\pi^2} = 0.25 \pm 0.18.$$

The 0.25 is the classical probability ($\frac{1}{2}\times\frac{1}{2}$). The symmetric state tends to have particles closer together, so there is a greater than normal probability of finding them on the same side; the antisymmetric state tends to separate particles. Symmetric (+sign) **0.43**, Antisymmetric (–sign) **0.07**.

8.41 There may be two in the $n = 1$ state, $E = 2\times\frac{1^2\pi^2\hbar^2}{2mL^2}$, two in the $n = 2$ state, $E = 2\times\frac{2^2\pi^2\hbar^2}{2mL^2}$, and the last would be forced into the $n = 3$ state, $\frac{3^2\pi^2\hbar^2}{2mL^2}$. Total $\mathbf{19\frac{\pi^2\hbar^2}{2mL^2}}$.

(b) Bosons do not obey an exclusion principle. All may be in the $n = 1$ state, $E = \mathbf{5\frac{\pi^2\hbar^2}{2mL^2}}$.

(c) With $s = 3/2$, there are four different possible value of m_s: –3/2, –1/2, +1/2, +3/2. Thus, without violation of the exclusion principle, four particles could have $n = 1$, with the fifth in the n = 2, $4\times\frac{1^2\pi^2\hbar^2}{2mL^2} + 1\times\frac{2^2\pi^2\hbar^2}{2mL^2}$ $= \mathbf{8\frac{\pi^2\hbar^2}{2mL^2}}$