Solution 3

Physics 313

4.19 Using result of exercise 18, $\lambda = \frac{h}{\sqrt{3mk_B T}} = \frac{6.63\times10^{-34}\,\text{J·s}}{\sqrt{3(9.11\times10^{-31}\,\text{kg})(1.38\times10^{-23}\,\text{J/K})(295\text{K})}} = \mathbf{6.29nm}$.

For a proton, $\lambda = \frac{6.63\times10^{-34}\,\text{J·s}}{\sqrt{3(1.67\times10^{-27}\,\text{kg})(1.38\times10^{-23}\,\text{J/K})(295\text{K})}} = \mathbf{0.147nm}$. Though the proton's speed would be smaller, its mass is so much larger that its momentum is much larger and wavelength smaller. In situations in which dimensions are comparable to or smaller than nanometers, the electron will exhibit its wave nature. At the same temperature, dimensions would have to be smaller by a factor of about forty for the proton to similarly exhibit its wave nature.

4.25 If its speed at one angstrom were greater than that for circular orbit, its orbit would not be a circle. It would at some other point reach *farther* from the proton than one angstrom. To find wavelength we need the speed for circular orbit. Assuming that it *does* behave as a classical particle, we use $F = ma$, where the electrostatic force gives the electron centripetal acceleration: $\frac{1}{4\pi\varepsilon_0}\left|\frac{(+e)(-e)}{r^2}\right| = m_e\frac{v^2}{r} \Rightarrow$

$$v^2 = \sqrt{\frac{1}{4\pi\varepsilon_0}\frac{e^2}{m_e r}} = \sqrt{8.99\times10^9\,\text{N·m}^2/\text{C}^2\,\frac{(1.6\times10^{-19}\,\text{C})^2}{(9.11\times10^{-31}\,\text{kg})(0.1\times10^{-9}\,\text{m})}} = 1.6\times10^6\,\text{m/s}.$$

The corresponding wavelength is thus, $\lambda = \frac{6.63\times10^{-34}\,\text{J·s}}{(9.11\times10^{-31}\,\text{kg})(1.6\times10^6\,\text{m/s})} \cong 4.6\times10^{-10}\,\text{m} = 0.46\text{nm}$. From this calculation we conclude that *at some point in its orbit* the electron's wavelength is well over 0.1nm, which is larger than the dimensions of the region where the electron moves. We see, then, that it **cannot be treated classically**.

4.30 For a massless particle, $E = pc$, or $hf = \frac{hc}{\lambda}$, so $\frac{E}{p} = c$.

(b) $\frac{(\gamma_u - 1)mc^2}{\gamma_u mu} = \left(1 - \frac{1}{\gamma_u}\right)\frac{c^2}{u}$.

(c) $\frac{\gamma_u mc^2}{\gamma_u mu} = \frac{c^2}{u}$.

(d) The quotients in (b) and (c) depend on the particle speed, which is a variable. For massless particles, the speed is not subject to variation.

(e) As $u \to c$, $\gamma_u \to \infty$ and both quotients approach c. The internal energy becomes negligible and the massive particle behaves more like a massless one.

4.43 $\Delta x\,\Delta p \geq \frac{1}{2}\hbar \to (5\times10^{-15}\,\text{m})(1.67\times10^{-27}\,\text{kg})\,\Delta v \geq \frac{1}{2}(1.055\times10^{-34}\,\text{J·s}) \Rightarrow \Delta v \geq \mathbf{6.3\times10^6\,m/s}$.
Its KE would be $\frac{1}{2}(1.67\times10^{-27}\,\text{kg})(6.3\times10^6\,\text{m/s})^2 \cong 0.2\text{MeV}$, on the low side of typical energies in the nucleus.

4.46 $\Delta x\,\Delta(mv) \geq \frac{1}{2}\hbar \to \Delta v \geq \frac{\hbar}{2m\Delta x} = \frac{\hbar}{2mL}$. The KE $\sim \frac{1}{2}m(\Delta v)^2 \geq \frac{\hbar^2}{\mathbf{8mL^2}}$

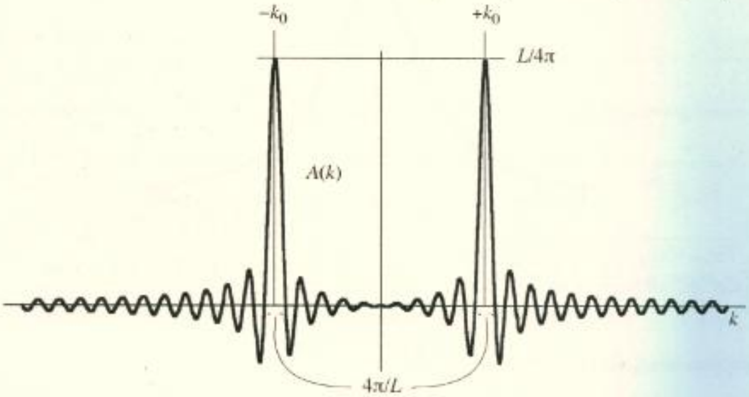
4.64 Given that $A(k) = \frac{C}{\pi}\frac{\sin(kw/2)}{k}$, $\psi(x) = \int_{-\infty}^{+\infty} \frac{C}{\pi}\frac{\sin(kw/2)}{k} e^{ikx}\, dk$. We know that $e^{ikx} = \cos kx + i \sin kx$. Because the rest of the integrand is an even function of k, the sin kx term will integrate to zero as the integral of an odd function over an interval symmetric about the origin. What remains is $\psi(x) = \int_{-\infty}^{+\infty} \frac{C}{\pi}\frac{\sin(kw/2)(\cos kx)}{k}\, dk$ = $\psi(x) = \frac{2C}{\pi}\int_{0}^{+\infty} \frac{\sin(kw/2)(\cos kx)}{k}\, dk$. Looking up the integral in a table of integrals shows its value to be 0 if $|x| > w/2$ and $\frac{\pi}{2}$ if $|x| < w/2$. Including the coefficient multiplying the integral we see that $\psi(x) = 0$ if $|x| > w/2$ and C if $|x| < w/2$. This is the original $\psi(x)$.

4.66 $$A(k) = \frac{1}{2\pi}\int_{-\frac{1}{2}L}^{+\frac{1}{2}L} \cos(k_0 x)e^{-ikx}\,dx = \frac{1}{2\pi}\int_{-\frac{1}{2}L}^{+\frac{1}{2}L} \frac{1}{2}\left(e^{+ik_0x} + e^{-ik_0x}\right)e^{-ikx}\,dx$$

$$= \frac{1}{4\pi}\int_{-\frac{1}{2}L}^{+\frac{1}{2}L}\left(e^{i(k_0-k)x} + e^{-i(k_0+k)x}\right)dx = \frac{1}{4\pi}\left(\frac{e^{i(k_0-k)\frac{1}{2}L} - e^{-i(k_0-k)\frac{1}{2}L}}{i(k_0-k)} + \frac{e^{-i(k_0+k)\frac{1}{2}L} - e^{i(k_0+k)\frac{1}{2}L}}{-i(k_0+k)}\right)$$

$$= \frac{1}{4\pi}\left(\frac{2i\sin((k_0-k)\frac{1}{2}L)}{i(k_0-k)} + \frac{-2i\sin((k_0+k)\frac{1}{2}L)}{-i(k_0+k)}\right) = \frac{\mathbf{L}}{4\pi}\frac{\mathbf{\sin((k_0-k)\frac{1}{2}L)}}{\mathbf{(k_0-k)\frac{1}{2}L}} + \frac{\mathbf{L}}{4\pi}\frac{\mathbf{\sin((k_0+k)\frac{1}{2}L)}}{\mathbf{(k_0+k)\frac{1}{2}L}}$$

A cosine is a sum of two complex exponentials, but because this cosine is spatially truncated, of width L, the complex exponentials have only approximate wave numbers $+k_0$ and $-k_0$. $A(k)$ is peaked at both k values but the peaks are not precise. Their widths are inversely proportional to L. Only if L is infinite would the peaks be of infinitesimal width; only then would $f(x)$ be a sum of two complex exponentials at precisely $+k_0$ and $-k_0$.



4.67 The question is: what wave number will never be measured? To answer this, find $A(k)$.

$$A(k) = \frac{1}{2\pi}\int_{-\frac{1}{2}w}^{+\frac{1}{2}w} Ce^{-ikx}\,dx = \frac{C}{2\pi}\frac{e^{-ik\frac{1}{2}w} - e^{+ik\frac{1}{2}w}}{-ik} = \frac{C}{\pi k}\frac{e^{+ikw/2} - e^{-ikw/2}}{2i} = \frac{C\sin(kw/2)}{\pi k}.$$

This is zero when $\frac{kw}{2} = n\pi$ or $k = \frac{2n\pi}{w}$, so that $p = \hbar k = \frac{\mathbf{2n\pi\hbar}}{\mathbf{w}}$ will never be found.