Solution 1

Physics 313

2.17 $\gamma_v \geq 1$. Classical mechanics applies when $v << c$, and $\gamma_v = 1$. At what speed will γ_v be 1.01?

$$\frac{1}{\sqrt{1-v^2/c^2}} = 1.01 \Rightarrow \mathbf{0.14c}$$

2.20 Your time is longer. $\Delta t_{\text{you}} = \gamma_v \, \Delta t_{\text{Carl}} \rightarrow 60\text{s} = \frac{1}{\sqrt{1-(0.5)^2}} \Delta t_{\text{Carl}} \Rightarrow \Delta t_{\text{Carl}} = \mathbf{52s}$

2.21 $L = L_0 \sqrt{1-v^2/c^2} \rightarrow 35\text{m} = L_0 \sqrt{1-(0.6)^2} \Rightarrow L_0 = \mathbf{43.75m}$

2.25 $\gamma_{0.8c} = \frac{1}{\sqrt{1-(0.8)^2}} = \frac{5}{3}$. Bob sees Anna's ship contracted to $100\text{m}/\gamma_v = 100\text{m}/\frac{5}{3} = 60\text{m}$, so Bob Jr. will have to be at x = **60m.**

(b) We seek t', knowing x, x', and t. $t' = \gamma_v\left(-\frac{v}{c^2}x + t\right) = \frac{5}{3}\left(-\frac{0.8}{3\times10^8\,\text{m/s}}(60\text{m}) + 0\right) = \mathbf{-2.67\times10^{-7}s.}$

2.26 Calling the front light Event 2, Anna frame S′, $t_2 - t_1 = \gamma_v\left(\frac{v}{c^2}\left(x_2' - x_1'\right) + \left(t_2' - t_1'\right)\right) = \gamma_v\left(\frac{v}{c^2}(60\text{m}) + 0\right)$. Since this is positive, the front time is the larger (later), so **back light must go on first.**

(b) $40\times10^{-9}\text{s} = \frac{1}{\sqrt{1-v^2/c^2}}\frac{v}{c^2}(60\text{m}) \rightarrow \left(1-v^2/c^2\right)(40\times10^{-9}\text{s})^2 c^2$

$= (v^2/c^2)\,(60\text{m})^2 \rightarrow (40\times10^{-9}\text{s})^2\,(3\times10^8\text{m/s})^2$

$= \left((60\text{m})^2 + (40\times10^{-9}\text{s})^2(3\times10^8\,\text{m/s})^2\right) v^2/c^2$

$\Rightarrow v/c = \mathbf{0.196}$

2.34 We may "work in either frame". *Muon's frame*: $\tau = 2.2\mu\text{s}$. Distance to Earth is shorter: $4\text{km}\,\sqrt{1-(0.93)^2}$ $= 1.47\text{km}$. $t = \frac{\text{dist}}{\text{speed}} = \frac{1{,}470\text{m}}{0.93\times3\times10^8\,\text{m/s}} = 5.27\times10^{-6}\text{s}$. $\frac{N}{N_0} = e^{-(5.27/2.2)} = e^{-2.4}$ or **9.1%.**

Earth frame: Distance to Earth is 4km. Lifetime is longer. $\frac{2.2\mu s}{\sqrt{1-(0.93)^2}} = 5.99\mu\text{s}$.

$$t = \frac{\text{dist}}{\text{speed}} = \frac{4{,}000\text{m}}{0.93\times3\times10^8\,\text{m/s}} = 1.43\times10^{-5}\text{s}. \quad \frac{N}{N_0} = e^{-(14.3/5.99)} = e^{-2.4} \text{ or } 9.1\%$$

(b) $\tau = 2.2\mu s$. $t = \frac{\text{dist}}{\text{speed}} = \frac{4{,}000\text{m}}{0.93\times3\times10^8\,\text{m/s}} = 1.43\times10^{-5}\text{s}$. $\frac{N}{N_0} = e^{-(14.3/2.2)} = e^{-6.5}$ or **0.14%**

2.40 We have a speed and time according to the lab and wish to know a distance according to that frame.

$$\text{distance} = (0.94 \times 3\times10^8\text{m/s})(0.032\times10^{-6}\text{s}) = \mathbf{9.02m.}$$

(b) If the experimenter sees $0.032\mu\text{s}$ pass on his own clock, he will see less pass on the clock glued to the particle.

$$\Delta t = \frac{\Delta t'}{\sqrt{1-(0.94)^2}} \rightarrow 0.032\mu s = \frac{\Delta t'}{\sqrt{1-(0.94)^2}} \Rightarrow \Delta t' = \mathbf{0.011\mu s.}$$

(c) Length contraction. According to the particle, the lab is $\sqrt{1-(0.94)^2}\,(9.02\text{m})$ = **3.08m** long. Let's see, if 3.08m of lab passes by in $0.011\mu\text{s}$, how fast is the lab moving? $\frac{3.08\text{m}}{0.011\times10^{-6}\text{s}} = .94c$. It all fits!