Physics 313
Final Exam
December 19, 2014

The exam is 3 hours in length.

There are ten problems. All are worth the same amount of points.

You should work any eight of the ten problems, your choice.

VERY IMPORTANT. If you work on more than eight problems, please clearly mark which ones you do not want me to grade. Best might be to draw an X trough them. If you work more than eight problems and don't indicate which are to be graded, I'll chose eight at random. I will NOT grade the eight best.

There is no extra credit for working more than eight problems

You may refer to two $8\frac{1}{2}'' \times 11''$ note sheets (double-sided) that you brought with you.

Best wishes for an enjoyable and peaceful semester break.

Problem 1

One thousand electrons are placed in a one-dimensional simple harmonic oscillator (SHO) potential well of frequency, ω. Note that for an SHO potential, the energy levels are equally spaced by $\hbar\omega$ with the energy of the nth level being $E_n = (n + 1/2)\hbar\omega$. That's all you need to know about the SHO energy levels.

a) What is the Fermi energy of the system?

b) What is the average electron energy?

c) What is the total energy?

d) Sketch the occupancy vs. n for the system at absolute zero, $T = 0$.

e) For a temperature T, what is the energy, in units of $k_B T$, of the state that has a probability of being 10% occupied?

Problem 2

Write down a correct Feynman diagram for each of the following:

a) Weak interaction process

b) QCD interaction process

c) QED interaction process

Problem 3

The Schwarzschild radius of a particle of mass M is: $r_S = \frac{2GM}{c^2}$

a) What is approximately the momentum of a probe particle, for example an electron, needed to probe down to a distance scale, r_S?

b) Assuming that the probe particle is extremely relativistic, what is its energy?

c) The effective gravitational mass of this probe particle is $M_{\text{eff}} = E/c^2$. If the distance being probed is equal to the Schwarzschild radius for this effective mass, then the amount of energy needed to probe to this distance, r_S, contained within a volume of this radius, produces a black hole. What is this energy at which a black hole will be produced?

Problem 4

Consider the $n = 3$ states of the hydrogen atom.

a) How many states are there with $n = 3$?

b) Label these states by giving their (l, m_l, m_s) quantum numbers.

c) Also, alternatively label the states by giving their (l, j, m_j) quantum numbers.

Problem 5

Muonic hydrogen occurs when the electron in hydrogen is replaced by its heavier cousin the muon. The muon has a mass that is 200 times greater than the mass of the electron.

a) What is the ground state energy of muonic hydrogen compared to the ground state energy of normal hydrogen.

b) What is the size of the ground state of muonic hydrogen compared to the size of the ground state of normal hydrogen.

c) If the ground state ($1s$ state) of muonic hydrogen is placed into a magnetic field, what will be the magnitude of the resulting level splitting compared to normal hydrogen.

Problem 6

a) List the matter particles of the Standard Model and indicate which of the three types of interactions (QED, QCD and Weak) they undergo.

b) List the gauge bosons of the Standard Model and indicate which of the three types of interactions (QED, QCD and Weak) they mediate.

c) In lecture, I listed nine fundamental shortcomings of the Standard Model. List any four of these.

Problem 7

Consider the states of the hydrogen atom with $n = 2$.

a) How many states are there?

b) For the case of no externally applied magnetic field, make a sketch of how these states are spilt in energy. The sketch does not need to be quantitative.

c) For the case of a strong externally applied magnetic field, make a sketch of how these states are spilt in energy. Strong means that the splitting is large compared to the spin-orbit splitting such that the spin-orbit coupling can be ignored. The sketch does not need to be quantitative.

d) For the case of a weak externally applied magnetic field, make a sketch of how these states are spilt in energy. Weak means that the splitting is small compared to the spin-orbit splitting. The sketch does not need to be quantitative.

Problem 8

Mary is driving her relativistic school bus. A student, Mike, is sitting in the rear of Mary's bus. In Mary and Mike's frame, the bus is 20 m long. Bob is standing at rest at the front door of a barn and Bill is standing at rest at the rear door of the same barn. In Bob and Bill's frame, the barn is 10 m long.

a) How fast must Mary be driving, such that Bob and Bill see the bus fit exactly inside the barn? That is, Bob sees the rear of the bus pass the front door at the same time as Bill sees the front of the bus pass the rear door.

b) What is the time difference between when Mary sees the front of the bus pass the rear door of the barn and Mike sees the rear of the bus pass the front door of the barn? Which happens first?

Problem 9

a) What is the wavelength of an electron with kinetic energy of 25 eV? Note that I'm giving you the electron's kinetic energy not its momentum. For the electron mass use 5×10^5 eV.

b) What is the width of the smallest one-dimensional box that can contain this electron?

c) If the electron is placed in a box of width $L = 1.0 \times 10^{-9}$ m $= 1.0$ nm, what would be the energy level (value of n) that the electron would be in? You might want to use $\hbar c = 197$ eV·nm.

Problem 10

a) If a gas of N non-relativistic, non-interacting, identical fermions is placed in a one-dimensional box of width L, what would be the force required to compress the box? Assume that the gas is in its ground state (T = 0). Note that the needed applied force is:

$$F = -\frac{dE}{dx}$$

b) If a gas of N relativistic, non-interacting, identical fermions is placed in a one-dimensional box of width L, what would be the force required to compress the box? Again take the gas to be in its ground state.