Quantum Critical Paraelectrics and the Casimir Effect in Time

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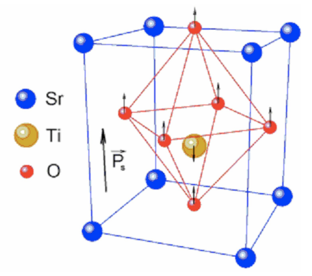
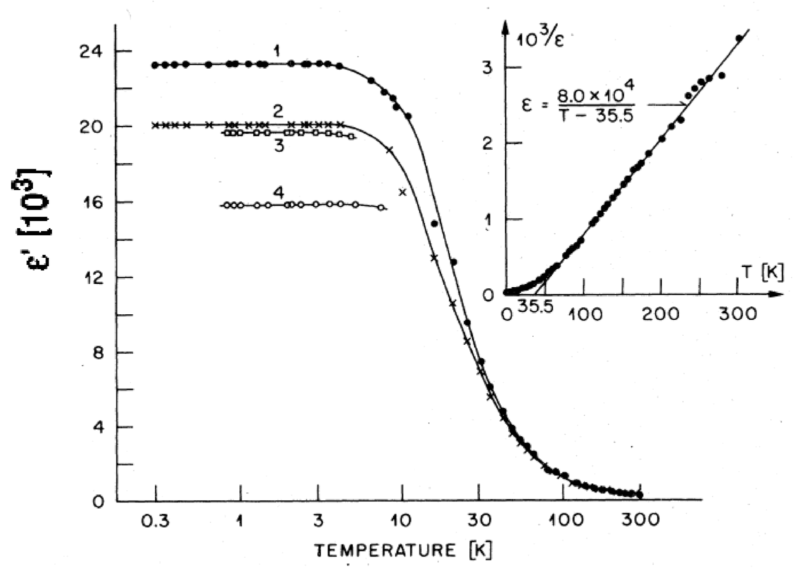
$SrTiO_3$: An Old Material in a (Cold) New Setting

- At low temperatures the perovskite $SrTiO_3$ displays a large dielectric response that saturates at T~4 K
- $SrTi^{16}O_3$ remains paraelectric down to T~0.3K

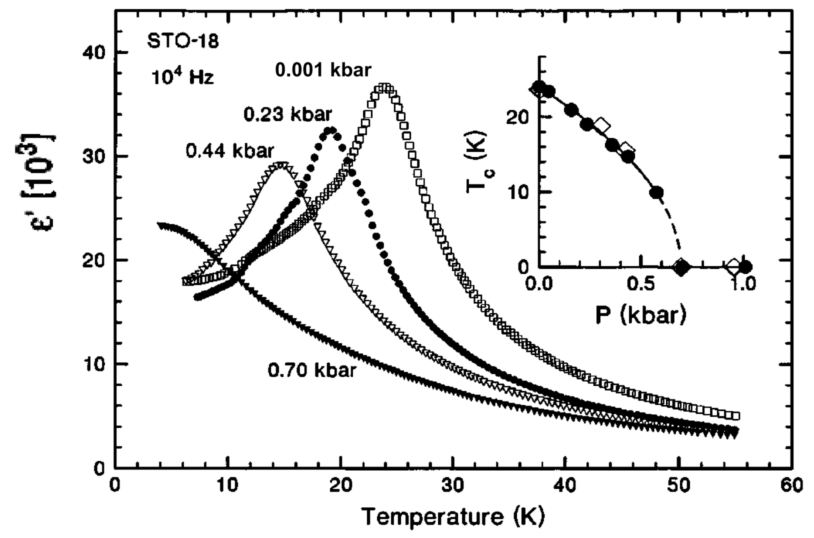


$SrTi^{16}O_3$: Quantum Paraelectric

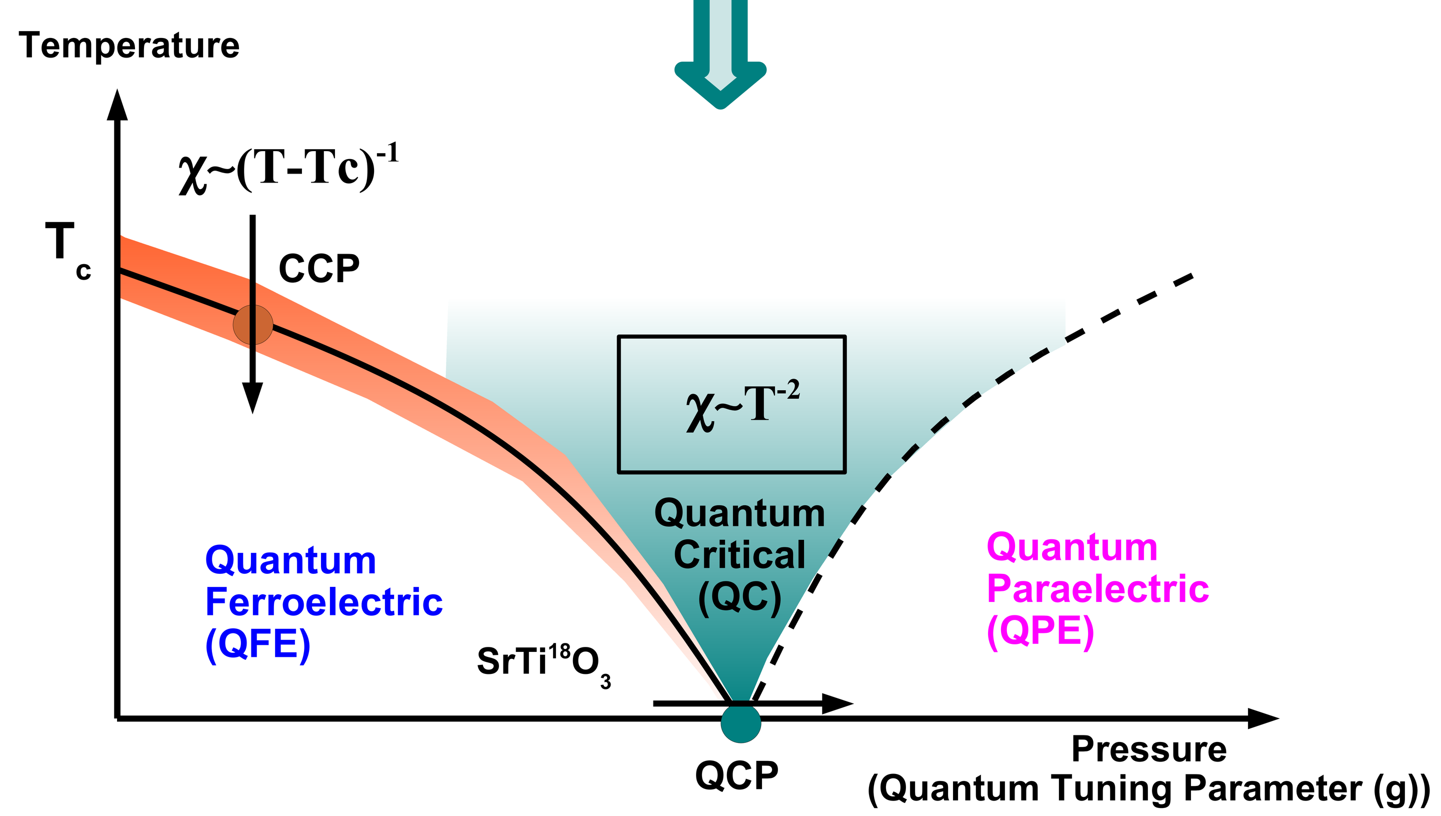


K.A. Müller and H. Burkard, PRB **19,** 3593 (1979)

$SrTi^{18}O_3$: Quantum Ferroelectric



E. L. Venturini, G. A. Samara, M. Itoh and R. Wang, Phys. Rev. B **69**, 184105 (2004)

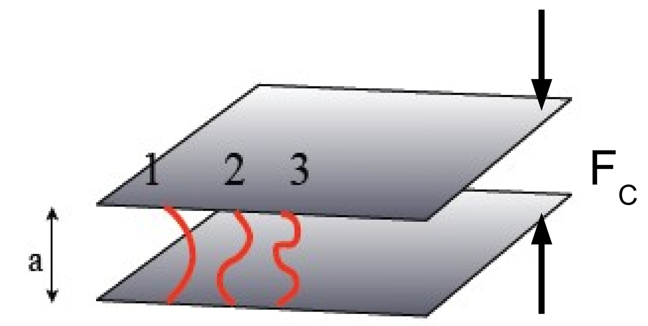


- We revisit and extend prior studies of the quantum paraelectric-ferroelectric transition within a modern framework of quantum critical theory

A.I. Larkin and D.E. Khmel'nitskii, Sov. Phys. JETP **29**, 1123 (1969)
A.B. Rechester, Sov. Phys. JETP **33**, 423 (1971)
D.E. Khmel'nitskii and V.L. Shneerson, Sov. Phys.- Solid State **13**, 687 (1971)
D.E. Khmel'nitskii and V.L. Shneerson, Sov. Phys. JETP **37**, 164 (1973)
J. A. Hertz, PRB **14**, 1165 (1973)
D. Schmeltzer, PRB **27**, 459 (1983)
R. Roussev and A.J. Millis, PRB **67**, 014105 (2003)

- Measurable predictions for thermodynamic response functions in the vicinity of the quantum ferroelectric critical point (QCP)
- $SrTiO_3$ is an attractive setting for the study of quantum criticality with detailed interplay between theory and experiment due to
  - the absence of electronic dissipation
  - negligible disorder
  - no competing fixed points

The Casimir Effect in Space and Time



$$a \Leftrightarrow \frac{1}{T}$$

- Neutral metallic planar structures attract each other on micrometer scales due to zero-point vacuum fluctuations
- Discrete modes of electromagnetic field between the plates

$$\omega_q = c\sqrt{q_\perp^2 + n^2\left(\frac{\pi}{a}\right)^2}\;;\quad q_n = \left(\frac{\pi}{a}\right)n$$

- Quantum criticality of the vacuum: plates remove zero modes, inducing finite correlation length

$$\xi = \frac{a}{\pi}$$

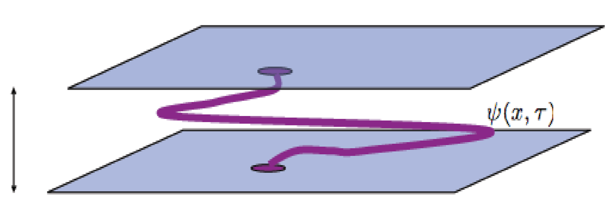
Coulomb interaction inside the cavity

$$V(q) \sim \langle \delta\phi_q \delta\phi_q \rangle = \frac{1}{q_\perp^2 + \xi^{-2}}$$

- Casimir vacuum energy

$$\frac{E_C}{A} = 2\sum_{n>0}\int \frac{d^2q}{(2\pi)^2}\frac{1}{2}\hbar\omega_q\;;\quad \frac{E_C - E_{C\,a\to\infty}}{A} = -\frac{\pi^2\hbar c}{720\,a^3}$$

$$\tau = \frac{\hbar}{k_B T}$$



$$Z = Tr[e^{-\beta H}] = \sum_{configs}\exp\{-\int_0^{\frac{\hbar}{k_B T}} d\tau\, d^d x\, L[P]\}$$

- Temperature acts as a boundary condition in time, giving rise to modes with discrete Matsubara frequencies

$$\nu_n = \frac{2\pi k_B T}{\hbar}\times n$$

- Quantum criticality near QCP: finite temperature removes modes, inducing finite correlation time

$$\xi_\tau = \frac{\hbar}{\kappa k_B T}$$

Fluctuation of the order parameter

$$\langle P_{q,\omega} P_{-q,-\omega}\rangle \sim \frac{1}{\omega^2 + c_s^2 q^2 + \xi_\tau^{-2}}$$

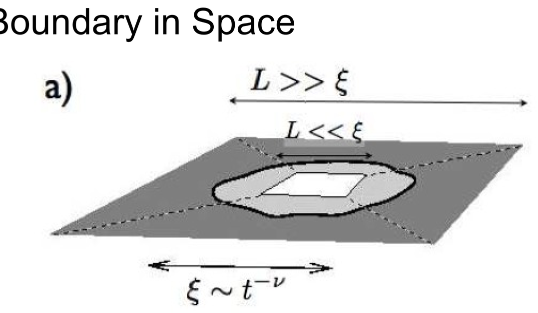
$$\chi = \langle P_{q,\omega}P_{-q,-\omega}\rangle_{\omega=q=0} \propto \frac{1}{T^2}$$

- Debye specific heat

$$c_V \sim \frac{d}{dT}\frac{E_C - E_{C\,T\to 0}}{V} \sim T^3$$

Boundary Effect in Space and in Time

Boundary in Space

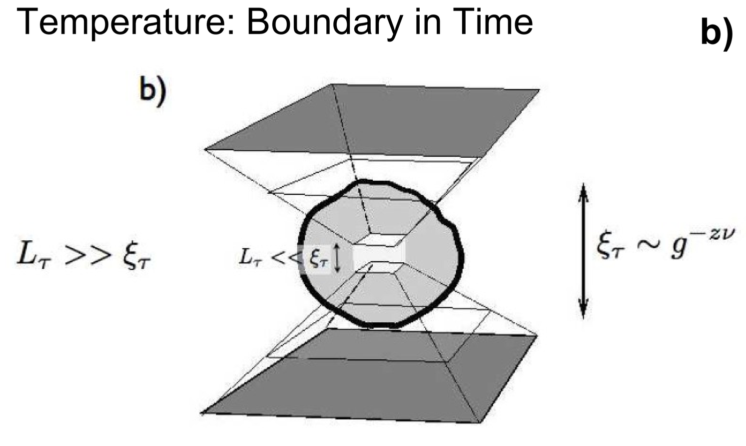


a) Classical Critical Point

$$\chi \sim t^{-\gamma}\Phi\left(\frac{L}{\xi}\right) \sim L^{\frac{\gamma}{\nu}}$$

Curie Law Boundary Effect

Temperature: Boundary in Time



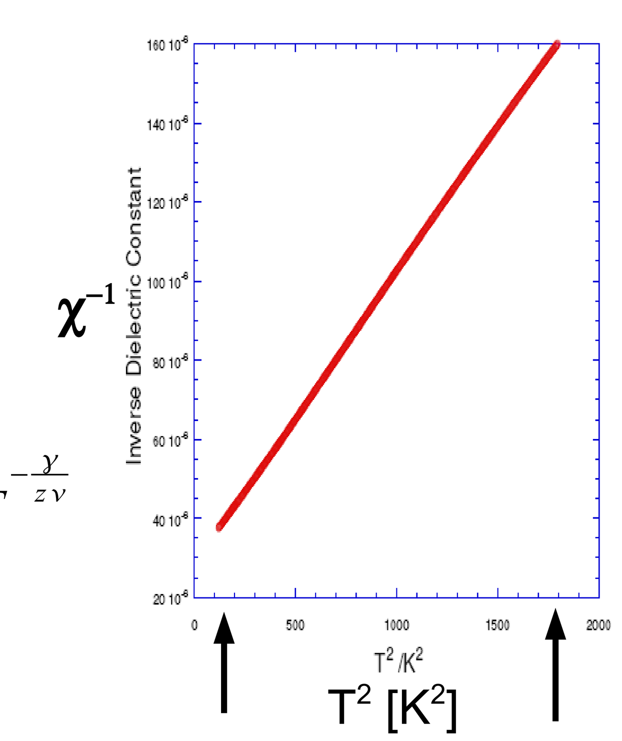
b) Quantum Critical Point

$$\chi \sim g^{-\gamma}\Phi\left(\frac{L_\tau}{\xi_\tau}\right) \sim L_\tau^{\frac{\gamma}{z\nu}} \sim T^{-\frac{\gamma}{z\nu}}$$

$$\chi \sim T^{-2}$$

$$g \sim T^2$$

$$\chi(\omega) \sim \frac{1}{\omega^2}F\left(\frac{\omega}{T}\right)$$

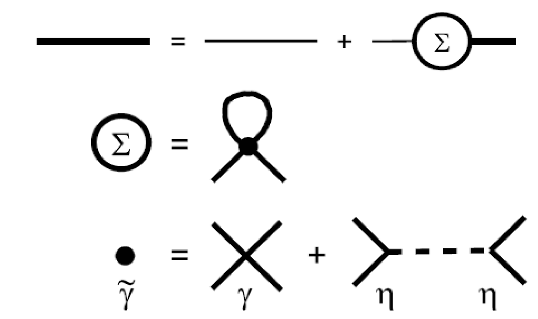


Rowley et al (unpublished)

Self-Consistent Hartree Theory

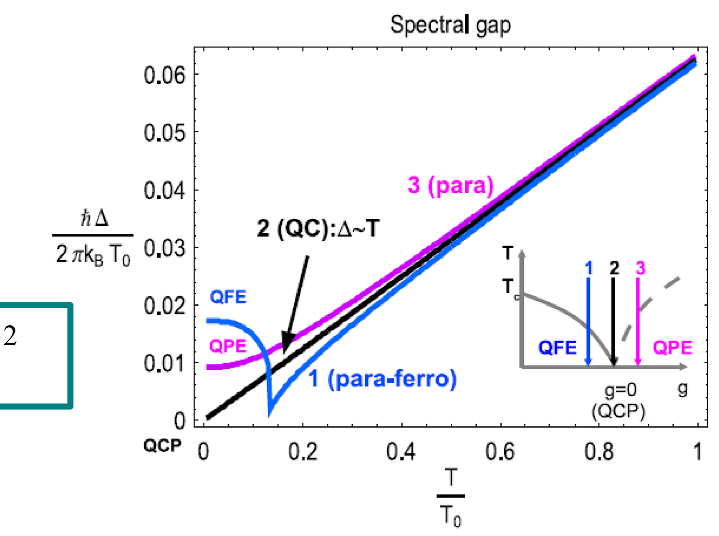
$$L_E[P,\Phi] = L_E[P] + \frac{m_a}{2}\left[(\partial_\tau\Phi)^2 + c_a^2(\nabla\Phi)^2\right] - \eta\nabla\Phi P^2$$

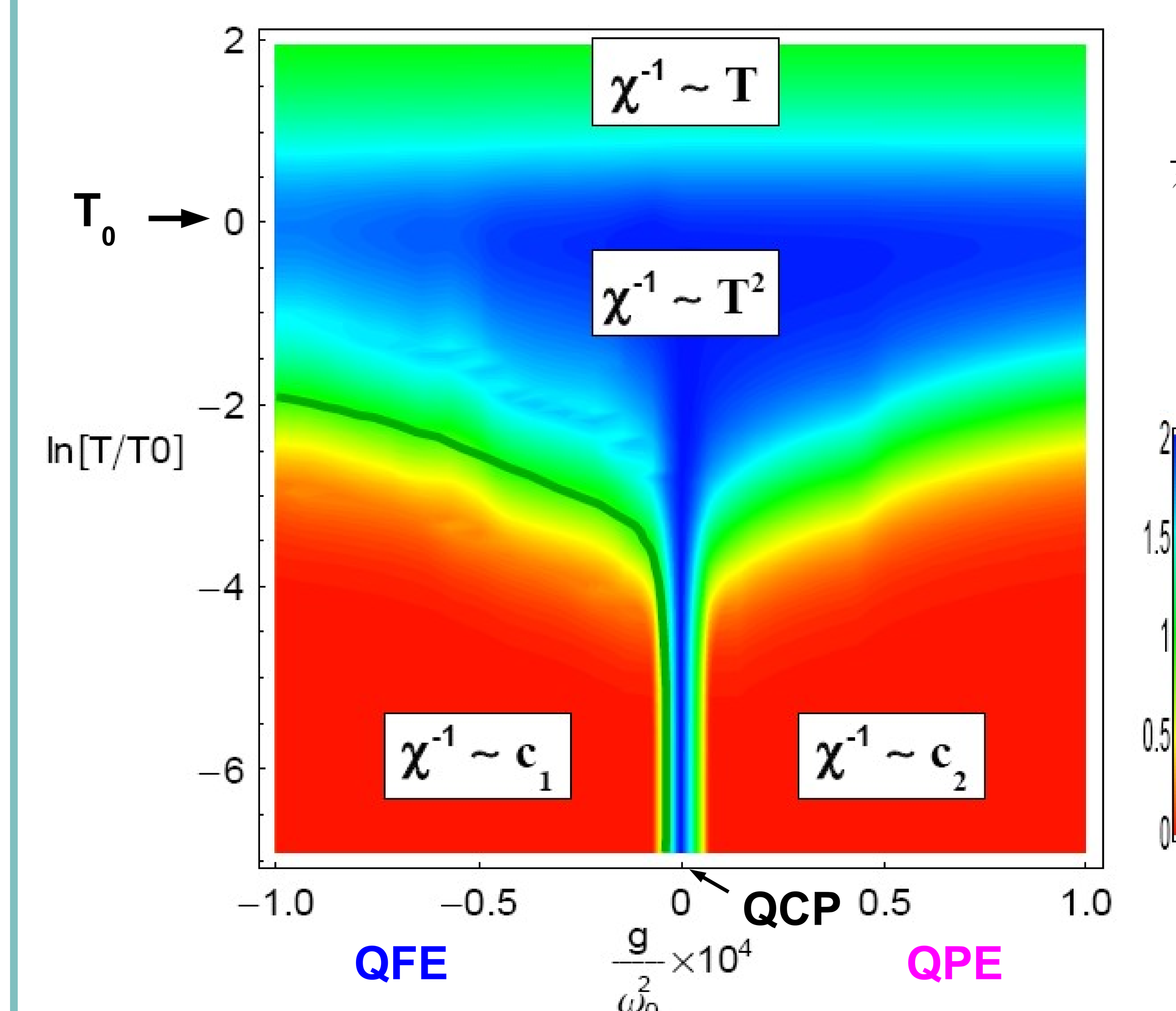
$$L_E = \frac{m}{2}\left[(\partial_\tau P)^2 + c_s^2(\nabla P)^2 + gP^2\right] + \frac{\gamma}{4}P^4$$

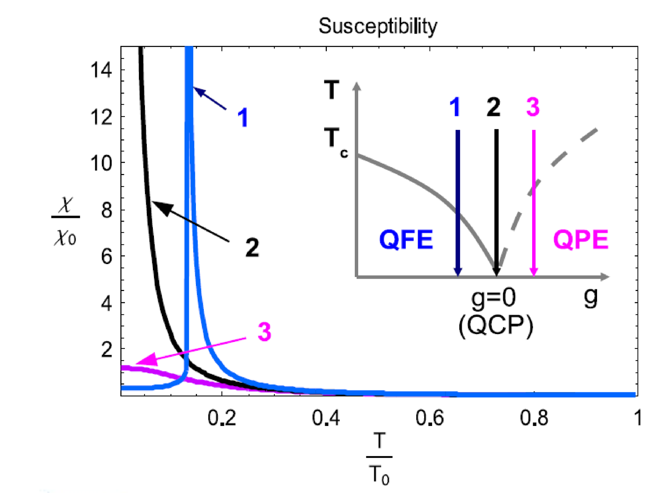


$$\omega(q) = \sqrt{c_s^2 q^2 + \Delta(T,g)^2}$$

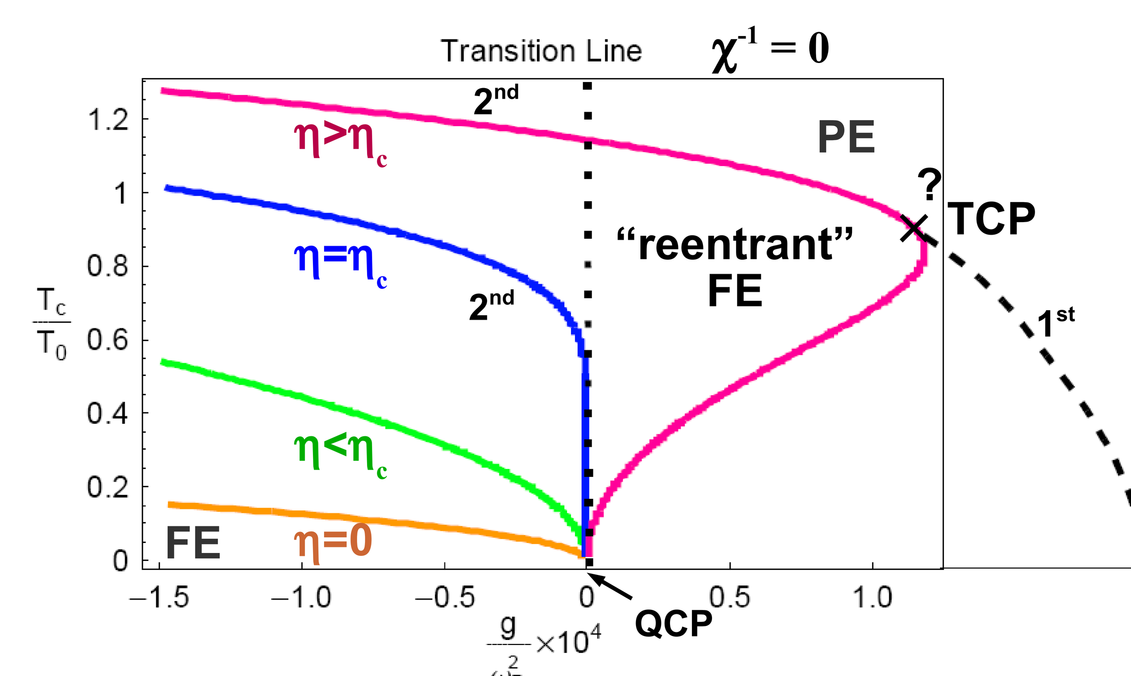
$$\chi(T) = \Delta(T,g)^{-2} \sim T^{-2}$$







Results without coupling to acoustic phonon (η=0)



- Strong coupling (η) to long-wavelength acoustic mode (Φ) leads to a reentrant quantum FE phase; possible phase coexistence
- Coexistence of FE and PE phase experimentally found in $SrTi^{18}O_3$ suggests: **η > ηc**

H. Taniguchi and M. Itoh, PRL **99**, 017602 (2007)

Open Questions

- Fluctuation-induced (generalized Larkin-Pikin) 1st order transition at T=0?
- Metaelectric Critical Point in the Pressure – Electric Field Plane?
- Doped $SrTiO_3$ at a QCP ? (add electrons) Superconducting $SrTiO_3$ at a QCP?
- Multiferroic QCP ? (add spins)