Comment on "Density of States near the Mott-Hubbard Transition in the Limit of Large Dimensions"

The Mott transition on the fully frustrated lattice with infinite coordination and semicircular density of states has been intensively studied by the groups at Rutgers and at ENS [1]. In our picture (which has by now been derived with many different techniques but still awaits a mathematically rigorous proof), it takes place, at $T = 0$, when the quasiparticle peak narrows continuously to zero at $U = U_{c2}$, while the insulating solution immediately above U_{c2} has a finite gap Δ. We note, however, that there is nonzero spectral weight at all frequencies in the metallic phase. These two properties can be expressed as follows, as the transition is reached from below:

(i) $\mathrm{Im}G(\omega, U) \neq 0$ for all $|\omega| < \Delta$ and for all $U < U_{c2}$.

(ii) $\lim_{U \to U_{c2}^-} \mathrm{Im}G(\omega, U) = 0$ for fixed ω, $0 < |\omega| < \Delta$.

A recent Letter [2] presents mathematical and physical arguments that claim to rule out a solution obeying (i) and (ii) under the assumption that the skeleton perturbative expansion (SPE) converges pointwise for $U < U_{c2}$. The SPE is indeed known not to converge in the paramagnetic Mott insulating state (PMI), i.e., for $U \geq U_{c2}$. Exactly at U_{c2}, the resonance has zero weight and therefore the system is in the PMI phase.

In this Comment we point out that Kehrein's mathematical argument is flawed by an incorrect exchange of orders of limits (even if one accepts Kehrein's hypothesis, i.e., the pointwise convergence of the skeleton expansion in the metallic phase $U < U_{c2}$). We also point out that Kehrein's physical argument is flawed by an incorrect interpretation of the physical meaning of the imaginary part of the self-energy in this problem.

The first part of Kehrein's paper, leading to his Eq. (9), proceeds along similar lines as Ref. [3]. It was shown there that the assumptions (i) and (ii) lead to a sharp resonance in the self-energy at a scale $\sqrt{w}\, t$ where $w \propto (U_{c2} - U)/U_{c2}$ is the quasiparticle residue. As a result the self-energy has considerable spectral weight in the gap, irrespective of the value of U:

$$\lim_{U \to U_{c2}^-} \int_{-\Delta}^{+\Delta} d\omega\, \mathrm{Im}\Sigma(\omega, U) \neq 0 . \tag{1}$$

The second part of Kehrein's paper uses the SPE and claims to show that assumptions (i) and (ii) imply

$$\lim_{U \to U_{c2}^-} \int_{-\Delta}^{+\Delta} d\omega\, \mathrm{Im}\Sigma(\omega, U) = 0 . \tag{2}$$

This would then contradict the correct statement, Eq. (1), hence showing the inconsistency of (i) and (ii). This part of Kehrein's work is, however, incorrect.

In Ref. [2], the self-energy is expanded in a SPE, i.e., as a power series in U in terms of the *fully interacting* Green's function as $\mathrm{Im}\Sigma(U, \omega) = \sum_{\alpha,m} U^m I_{\alpha,m}(\omega)$, where α labels each graph of the SPE, at order m. Using Cutkosky rules and (i) and (ii) *for each given graph* in the SPE $\int_{-\Delta}^{+\Delta} d\omega\, I_{\alpha,m}(\omega)$ is found to be of order wt^2 and hence vanishes as $U \to U_{c2}^-$. This graph by graph estimate, leads to Kehrein's Eq. (12): $\int_{-\Delta}^{+\Delta} d\omega\, I_{\alpha,m}(\omega) = \tilde{\gamma}_n (w/2)^{2n+1}$ with $\tilde{\gamma}_n \approx \alpha_n / w^{2n}$. To arrive at Eq. (2) Kehrein *interchanges the limit $U \to U_{c2}^-$ and the resummation of the SPE*. This incorrect exchange of limits is responsible for the incorrect conclusions in Ref. [2]. Pointwise convergence and uniform bounds *on the coefficient of a series* do not justify the exchange of order of limits. *Uniform* convergence in the closed interval ending at U_{c2} is actually needed, which is precluded here by the fact that the SPE is known to diverge at U_{c2}.

Finally, we comment on Kehrein's physical arguments. He states that "In the skeleton expansion, the imaginary part of the self-energy is related to the available phase space for scattering processes." Since the density of states is vanishingly small near the Mott transition, a large value of the imaginary part of the self-energy (interpreted as a scattering rate) would not seem, at first sight, internally consistent. The error in this argument is that the resonance [3] in the self-energy occurs at a scale $\sqrt{w}\, t \gg wt$ which is *outside the region of validity of Fermi liquid theory*. It is thus totally unrelated to quasiparticle scattering. This resonance is the precursor of the pole in the PMI. Above the scale wt in the metal, or in the PMI, one should not interpret the self-energy in terms of quasiparticles.

The derivation of the dynamical-mean field equations in the large-d limit *does not require* the use of the SPE (see, e.g., the "cavity" construction in Ref. [1]). The SPE has not yet given any useful results about the Mott transition, which led us to develop alternative, more successful, nonperturbative methods [1,4].

In summary, Kehrein's conclusions are based on mathematical and physical errors. They have no bearing on the validity of our results on the Mott transition.

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