

THE MAGNETIC FIELD CROSSOVER EXPONENT PROBLEM REVISITED

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The magnetic field crossover problem is reexamined in the context of the nonlinear σ model for non-interacting electrons in a disordered system. By using a Wilson–Polyakov renormalization group, the orbital magnetic field crossover exponent is found to agree to lowest order with the exponent expected from naive scaling arguments. It is also argued that this result is in fact valid to all orders.

In the presence of a magnetic field B the Anderson Hamiltonian [1] for a disordered electron gas is given by

$$H = \frac{1}{2m} \left(\frac{\nabla}{i} - \frac{e}{c} A \right)^2 + V(r) - g\mu_B \hat{\sigma} \cdot B, \quad (1)$$

where $A(r)$ is the vector potential and $V(r)$ is the impurity potential. g and μ_B are the Landé g -factor and Bohr magneton respectively and $\hat{\sigma}$ is the spin operator. The magnetic field introduces two terms in addition to $H_0 = -\nabla^2/2m + V(r)$. $H_1 = (e/2imc) (\nabla_\mu A_\mu + A_\mu \nabla_\mu) + (e^2/2mc^2) A^2$ couples to the electronic orbital current, while $H_2 = -g\mu_B \hat{\sigma} \cdot B$ couples to the spin of the electrons. They induce two characteristic energy scales

$$\Omega_1 \sim v_F^2 \tau \frac{eB}{\hbar} \sim E_F \tau \frac{eB}{m\hbar}, \quad \Omega_2 \sim \frac{eB}{m\hbar},$$

where τ is the scattering time in the Born approximation and v_F and E_F are the Fermi velocity and the Fermi energy respectively. For weak disorder $\Omega_1 \gg \Omega_2$, which makes orbital effects easily observed in experiments.

After the realization that the orbital magnetic coupling affects the quantum interference responsible for orthogonal localization, the issue of the orbital magnetic field crossover exponent at the orthogonal fixed point was raised and answered in a very physical paper [2] by Khmel'nitsky and Larkin (KL). Close to the Anderson transition the microscopic parameters enter the physics via the

divergence of the localization length $\xi = (E - E_c)^{-\nu}$, E_c being the mobility edge. KL argued that the magnetic field introduces a new length $L_B = \sqrt{\hbar c/eB}$ and that the magnetic field dependence in physical quantities should come via the total magnetic flux Φ enclosed in a region of size ξ , $\Phi = (e/\hbar c) B \xi^2 = (\xi/L_B)^2$. For the conductivity σ they then wrote

$$\sigma = \xi^{-(d-2)} f\left(\frac{L_B}{\xi}\right) = (E - E_c)^{(d-2)\nu} g\left(\frac{B}{(E - E_c)^{2\nu}}\right), \quad (2)$$

showing that the crossover exponent for B is

$$\varphi_B = 2\nu \quad (3)$$

exactly.

There have been many attempts to give a satisfactory microscopic basis to KL phenomenological analysis using field theory [3, 4]. The term H_1 breaks time reversal symmetry (I) but is spin conserving. H_2 breaks the spin rotational symmetry (II). These symmetries can also be broken by suitable impurity scattering. Oppermann [3] and later Wegner [5] analyzed this latter problem in the context of the $\varepsilon = d - 2$ expansion. It was found that when symmetry I is broken but symmetry II is unbroken the crossover exponent is given by

$$\varphi_a = \nu[2 + \frac{9}{4}\zeta(3)\varepsilon^4 + \mathcal{O}(\varepsilon^5)], \quad (4)$$

while if both symmetries are broken the crossover exponent is given by

$$\varphi_s = \nu[2 + 2\varepsilon - 3\zeta(3)\varepsilon^4 + \mathcal{O}(\varepsilon^5)]. \quad (5)$$

If one accepts the naive idea that all perturbation which break the same set of symmetries are physically equivalent, the results in eqs. (2) and (4) are contradictory.

Houghton, McKane and Cerdeira [6] argued that a magnetic field, unlike potential perturbations, cannot be described in terms of a symmetry breaking mass term and suggested a more complicated scenario.

Here [7] we resolve this puzzle by deriving microscopically the operators induced in the field theory by the magnetic field and calculating their renormalization to one loop order in the framework of the Wilson–Polyakov renormalization group approach [8, 9]. Our field theoretical arguments prove the validity of the KL scaling analysis. It turns out that the break of time reversal by a magnetic field is described by different operators than the scattering by a symmetry breaking potential. It is also argued that the KL crossover exponent $\varphi_B = 2\nu$ is in fact valid to all orders in ε .

Starting from the Anderson Hamiltonian, eq. (1), we obtain a nonlinear σ model describing the interactions of the soft modes following Efetov, Larkin and Khmel'nitsky [9]. This model is represented in terms of matrices with replica ($a = 1, n$), spinor ($\sigma = 1, 2$) and energy ($p = 1, 2$) indices,

$$Q = T\hat{s}T^{-1}, \quad \hat{s} = \delta_{aa'}\delta_{\sigma\sigma'}\delta_{pp'}(-1)^{p+1}, \quad (6)$$

with $T \in \text{Sp}(4n)$. The matrices Q can be written as $2n \times 2n$ quaternion matrices obeying the relationships $Q^2 = I$, $Q = Q^\dagger$ and $C^T Q^T C = Q^\dagger$, where $C = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ is the charge conjugation matrix in the spinor space and superscript T indicates the transposition operation.

The nonlinear σ model Lagrangian $L[Q]$ reads

$$L[Q] = L_{xx} + L_A + L_{A^2} + L_{xy}, \quad (7)$$

$$L_{xx} = \frac{\sigma_{xx}}{16} \int d^d r \text{Tr}\{(\nabla Q)^2\}, \quad (8)$$

$$L_A = -i \frac{\sigma_{xx} e}{8c} \int d^d r A_\mu \text{Tr}\{\nabla_\mu Q[Q, \sigma_3]\}, \quad (9)$$

$$L_{A^2} = -\frac{\sigma_{xx} e^2}{16c^2} \int d^d r A^2 \text{Tr}\{[Q, \sigma_3]^2\}, \quad (10)$$

$$L_{xy} = \frac{\sigma_{xy}}{16} \int d^d r \varepsilon_{\mu\nu} \text{Tr}\{Q \nabla_\mu Q \nabla_\nu Q \sigma_3\}. \quad (11)$$

$e^2 \sigma_{xx} / 2\pi = ne^2 \tau / m$ is the Drude conductivity, n here being the electron density, and $e^2 \sigma_{xy} / 2\pi = (ne^3 \tau^2 / m^2 c) B$ is the Born formula for the Hall conductivity. The terms (9) and (10) were first derived in ref. [9]. The term (11) generalizes to the present orthogonal case the topological term considered by Pruisken [10] in the unitary limit. In eq. (7) L_{xx} is invariant under $\text{Sp}(2n) \times \text{Sp}(2n)$. The operators L_A , L_{A^2} and L_{xy} lower the symmetry to $\text{U}(n) \times \text{U}(n)$.

In the absence of magnetic field L_{xx} is multiplicatively renormalizable and under a renormalization group transformation the charge σ_{xx} obeys the recursion relation [9]

$$\sigma'_{xx} = b^{d-2} \sigma_{xx} \left(1 - \frac{1+2n}{\pi \sigma_{xx}} \log b\right), \quad (12)$$

where b is the rescaling factor. In the limit of zero replica, eq. (12) yields a fixed point $\sigma_{xx} = i / \pi \varepsilon$ and a critical exponent $1/\nu = \varepsilon + \mathcal{O}(\varepsilon^2)$.

We turn now to the crossover analysis. Notice that L_A has bare dimensions $d-1$ while L_{A^2} has bare dimension d . The vector field A has therefore dimensions $d-1$ in L_A and it has dimensions $d/2$ in L_{A^2} . These two terms are equally relevant at $d=2$ and both have to be considered in the crossover analysis. By applying the Wilson-Polyakov renormalization procedure to L_A we find that L_A is multiplicatively renormalizable and under renormalization it transforms according to

$$L'_A = b^{d-1} \left(1 - \frac{1+2n}{\pi \sigma_{xx}} \log b\right) L_A. \quad (13)$$

Similar, we get for L_{A^2} in the zero replica limit

$$L'_{A^2} = b^d \left(1 - \frac{1}{\pi \sigma_{xx}} \log b\right) L_{A^2}. \quad (14)$$

For finite replica L_{A^2} is not renormalizable. In fact it contains the antisymmetric operator which is induced in ref. [5] by the time reversal invariance breaking scattering. However the combination $L_A + L_{A^2}$ happens to be renormalizable when the recursion relations for the two operators are carried out to second order in A . That is, provided the contribution from $(L_A)^2$ is included in L'_{A^2} , this operator has the same structure as L_{A^2} and we get

$$L'_{A^2} = b^d \left(1 - \frac{1+2n}{\pi \sigma_{xx}} \log b\right) L_{A^2}. \quad (15)$$

From eqs. (13) and (15) we obtain the following relations:

$$\sigma'_{xx} A'(r) = b^{d-1} \left(1 - \frac{1+2n}{\pi \sigma_{xx}} \log b\right) \sigma_{xx} A(br), \quad (13')$$

$$\sigma'_{xx} A'^2(r) = b^d \left(1 - \frac{1+2n}{\pi \sigma_{xx}} \log b\right) \sigma_{xx} A^2(br), \quad (15')$$

which combined with the recursion relation for σ_{xx} , eq. (12), both lead to

$$A'(r) = b A(br). \quad (16)$$

A scales therefore with its bare dimension. Notice that eqs. (13) and (15) together with $L'_{xx} = (\sigma'_{xx} / \sigma_{xx}) L_{xx}$ sum into

$$L'_{xx} + L'_A + L'_{A^2} = b^{d-2} \left(1 - \frac{1+2n}{\pi \sigma_{xx}} \log b\right) (L_{xx} + L_A + L_{A^2}) \quad (17)$$

so as to implement gauge invariance. We comment on that below.

For a static field $A_x(r) = -(e^{iqr}/2iq_y)B$, $A_y(r) = (e^{iqr}/2iq_x)B$ one finds from eq. (16) in the limit $q \rightarrow 0$

$$B' = b^2 B, \quad (18)$$

which implies the crossover exponent $\varphi_B = 2\nu$ as stated in eq. (3). This is the microscopic justification of the KL scaling analysis.

While so far the renormalization group analysis has been carried out to lowest order in ε , we now argue that $\varphi_B = 2\nu$ holds for all orders in the ε expansion. From our analysis it follows that we should consider the operator

$$L_{xx} + L_A + L_{A^2} = \frac{1}{i6} \sigma_{xx} \int d^d r \text{Tr} \left\{ \nabla Q - i \frac{e}{c} A[Q, \sigma_3] \right\}^2 \quad (19)$$

as one unit. This operator is invariant under a gauge transformation

$$T \rightarrow e^{i\varphi(r)\sigma_3} T, \quad A \rightarrow A - \frac{c}{e} \nabla \varphi. \quad (20)$$

Since the renormalization procedure respects that symmetry, the anomalous dimensions of L_A and L_{A^2} must equal those of L_{xx} to all order in ε . Therefore there is only one renormalization constant in the theory and the field A transforms according to its naive dimension which results in eq. (16).

The renormalization of the operator L_{xy} is more intriguing. We have shown that it is multiplicative renormalization for finite replica and its coefficient σ_{xy} obeys the relation

$$\sigma'_{xy} = b^{d-2} \sigma_{xy} \left(1 - \frac{1+2n}{\pi \sigma_{xx}} \log b \right). \quad (21)$$

In the unitary limit L_{xy} is not renormalized perturbatively because it is a topological term. The fact that for orthogonal symmetry we find it is perturbatively renormalized is related to the trivial topology of $\text{Sp}(4n)/\text{Sp}(2n) \times \text{Sp}(2n)$. Linearizing eq. (21) around the fixed point $1/\pi \sigma_{xx} = \varepsilon$ we find a crossover exponent for this operator,

$$\varphi_{xy} = 0. \quad (22)$$

If we identify the critical behaviour of the coupling σ_{xy} with the critical behaviour of the Hall conductivity we would find that

$$\sigma_{xy} \sim (E - E_c)^{\nu\tau} \quad (23)$$

near the mobility edge and therefore the ratio $\sigma_{xy}/\sigma_{xx}^2$ would diverge at the mobility edge. This could be interpreted physically if we assume that the Hall number measures the number of mobile carriers with no contributions coming from the localized states. However notice that eq. (23) is in sharp disagreement with lowest order perturbation theory results which predict that the Hall number is unrenormalized by localization effects [11]. The interpretation of the perturbative series for the Hall conductivity in terms of scaling operators in the nonlinear σ model happens to be rather involved and will be subject to a future work.

For completeness we finally comment on the renormalization of the spin-dependent term generated in $L(Q)$ by the Zeeman coupling H_2 ,

$$L_{\text{spin}}[Q] \propto ig\mu_B B \text{Tr} \{ \sigma_3 \tau_3 Q \}. \quad (24)$$

Here σ_3 and τ_3 are intended to act in the spin and spinor space respectively. No anomalous dimensions are found to L_{spin} in the zero replica limit. The renormalization of L_{spin} has in fact to equal the renormalization of the spin susceptibility [12] which stays finite at the localization transition in the absence of interelectronic forces. Therefore L_{spin} scales as its bare dimensions d . However, unlike the case of spin dependent scattering (or in the presence of electronic interactions), the reduction of symmetry induced by L_{spin} does not affect the conductivity. Because of the opposite shifts in the mobility edge for spin up and down, adding a field at $E_F = E_c$ ($B = 0$) we expect $\sigma \sim B^{(d-2)\nu}$. The crossover behaviour governed by the exponent $\varphi_{\text{spin}} = \nu d$ could be instead observed in the transverse spin diffusion, $\sigma_{\text{spin-L}} \sim B^{(d-2)/d}$, provided the electron-electron interactions are negligible, which, however, is usually not the case.

References

- [1] For a general review see: Anderson Localization, Springer Series in Solid-State Sciences, vol. 39, Y. Nagaoka and H. Fukuyama, eds. (Springer, Berlin, 1982).
- [2] D.E. Khmel'nitsky and A.I. Larkin, Solid State Commun. 39 (1981) 1069.
- [3] R. Oppermann, J. Phys. (Paris) 45 (1984) L1161.
- [4] D. Belitz, Solid State Commun. 52 (1984) 989.
- [5] F. Wegner, Nucl. Phys. B270 (1986) 1; F. Wegner, Nucl. Phys. B316 (1989) 663.
- [6] A. Houghton, J.J. McKane and H.A. Cerdeira, J. Stat. Phys. 48 (1987) 81.
- [7] A fuller version of this work can be found in: M. Biafore, C. Castellani and G. Kotliar, Nucl. Phys. B340 (1990), in press.
- [8] A.M. Polyakov, Phys. Lett. B 59 (1975) 19.

- [9] K.B. Efetov, A.I. Larkin and D.E. Khmel'nitsky, *Zh. Eksp. Teor. Fiz* 79 (1979) 1120 [*Sov. Phys. JETP* 52 (1980) 568].
- [10] A.M.M. Pruisken *Nucl. Phys.* B235 (1984) 277; A.M.M. Pruisken, Field theory, scaling and the localization problem, in: *The Quantum Hall Effect*, R.E. Prange and S.M. Girvin, eds. (Springer, New York, 1987) p. 117.
- [11] H. Fukuyama, *J. Phys. Soc. Jpn.* 49 (1980) 644.
- [12] R. Raimondi, C. Castellani and C.Di Castro, preprint (1989).

ANDERSON LOCALIZATION PROBLEMS IN GAPLESS SUPERCONDUCTING PHASES

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The interplay of Anderson localization and different kinds of superconducting order is most interesting in "gapless" cases, i.e. for nonvanishing electron density of states at E_F . I present a new renormalization group result for Anderson localization in the gapless type II limit of an Ising superconducting (SC) glass. From this calculation a guess is also made for the XY superconducting glass. In both cases, and in contrast to localization in normal systems, two renormalization constants (one for field- and one for coupling constant renormalization) are necessary (and sufficient). The density of states at E_F is singular with exponent β . For the Ising SC-glass I obtain $\nu^I = 1/(d-2)$, $\beta^I = 1/2$, and $\eta^I = 0$, while the XY SC-glass has $\nu^{XY} = 1/(d-2)$, $\beta^{XY} = 1$, and $\eta^{XY} = d-2$, all in leading order of the $d-2$ expansion and for $E = E_F$. For $E \neq E_F$ a symmetry argument, and also the calculation given here, predict usual localization behaviour with $\nu = 1/(d-2)$, $\beta = 0$, and $\eta = 2-d$ in both cases.

The effects of Cooper pairs on localization in the pure superconducting glasses is compared with earlier results showing perfect coexistence of Anderson localization with dirty superconductivity approximately described by a nonrandom order parameter. These limiting cases are embedded in a more general field theory given here, which contains three superconducting order parameters and a conventional pair-breaking mechanism.

1. Introduction

The first part of this report reviews briefly aspects of the gauge invariant local pairing theory of dirty type II superconductors. The important role of Edwards-Anderson (EA) type superconducting glass order parameters both for localization of electron states and for the behaviour of the composite particles (Cooper pairs) is emphasized. The perfect coexistence of idealized nonrandom superconducting order in a random system with Anderson localization due to disorder is contrasted, in the second part of this article, with renormalization group results showing new classes of critical localization behaviour in gapless superconducting glasses as a consequence of singular behaviour in the electron density of states at E_F .

In the first case, time reversal invariance was supposed, whence the gap in the electron density of states was equal to the homogeneous superconducting order parameter. Bogoliubov diagonalization mapped the superconducting nonlinear σ model onto the compact symplectic nonlinear σ model for normal