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Conductivity and Tunnelling Density of States Exponents at the Metal Insulator Transition with Strong Spin Orbit Scattering

INTRODUCTION

Our understanding of the metal insulator transition in interacting disordered Fermi systems has increased enormously recently.[1] The mapping of the metal-insulator transition problem in the presence of randomness and electron-electron interactions into a generalized nonlinear σ-model[2] allowed the identification of the scaling variables and provided an efficient tool for generating a perturbation theory in powers of the dimensionless resistance $t \equiv \frac{e^2}{2\pi^2\hbar} R$. The results of the nonlinear σ-model have been shown to be consistent with diagrammatic perturbation theory satisfying all the Ward identities deriving from the global gauge invariance of the model.[3] The scaling variables relevant at the metal-insulator transition have been interpreted in terms of quasi-particle parameters in a Landau-like framework.[4]

There are several universality classes describing different physical scenarios for a metal-insulator transition.[5] The different universality classes are determined by the conservation laws, the range of the electron-electron interactions and by the symmetries, which in turn depend on whether the system has magnetic impurities, spin orbit scattering, or is in the presence of strong or weak magnetic fields.

In this paper we reconsider the metal-insulator transition in the presence of spin orbit scattering and electron-electron interaction. We provide a detailed derivation of the renormalization group (RG) equations first derived by Castellani et al.[6] We use a parametrization which makes the calculations more similar to the field theory calculations in more conventional σ models.[8] This parametrization allows us to calculate the singular corrections to the one particle density of states and its critical index to one loop order.[7]

The only detailed measurements of the tunnelling density of state close to the metal insulator transition have been carried out in systems having strong spin orbit scattering[9,10] and we compare the results of the ϵ expansion with the experiments.

The content of the paper is the following: in Section 1 we outline the derivation of the renormalization group equations in the presence of spin orbit scattering. For pedagogical reasons we present a detailed derivation of the perturbation theory of the nonlinear σ model describing the diffusion modes of interacting electrons. Section 2 contains the evaluation of the critical index ν to order $O(\epsilon^{3/2})$. In Section 3 we discuss the one particle density of states corrections. We conclude in Section 4 with a comparison of the theoretical predictions and the results of tunneling experiments.

I. THE PERTURBATION THEORY AND THE RENORMALIZATION GROUP EQUATIONS

The starting point is the field theoretical formulation of the interacting disordered Fermi system due to Finkel'stein.[2] The partition function of the system is given by a functional integral over a matrix field Q to be specified below.

$$Z = \int DQ e^{-(L_d[Q] + L_{ee}[Q])} \tag{1}$$

The Lagrangian

$$\begin{aligned} L_d[Q] &= \frac{1}{8t}\int dr tr \nabla Q \nabla Q - \frac{z}{2t}\int tr \hat{\epsilon} Q \\ L_{ee} &= -\frac{1}{2t}\int dr \left[\Gamma_s \sum_{a=0,3} Q^a \gamma_d Q^a (-1)^a + \frac{\Gamma_c}{2}\sum_{a=1,2} Q^a \gamma_c Q^a\right] \end{aligned} \tag{2}$$

contains four parameters: t is the inverse diffusion coefficient, z is related to the renormalization of the specific heat,[5] Γ_s is the singlet scattering amplitude, Γ_c is the scattering amplitude in the Cooper channel. These scaling parameters have been recently reinterpreted in terms of a Fermi liquid framework for a disordered system.[5] $\hat{\epsilon}$ is a matrix in the frequency space, replica space and is proportional to the identity quaternion $\hat{\epsilon}_{nm} = \delta_{nm}\epsilon_n\delta_{ij}\tau_0$, ϵ_n is a Matsubara frequency, $(2n +$

$1)\pi T$. Q is a matrix in frequency space, and replica indices. Each matrix element is a quaternion of the form

$$Q_{ni,mj} = Q^0_{ni,mj}\tau_0 + iQ^1_{ni,mj}\tau_1 + iQ^2_{ni,mj}\tau_2 + Q^3_{ni,mj}\tau_3 \tag{3}$$

with

$$\tau_0 = \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} \quad \tau_1 = \begin{pmatrix}0 & -i\\ -i & 0\end{pmatrix} \quad \tau_2 = \begin{pmatrix}0 & -1\\ 1 & 0\end{pmatrix} \quad \tau_3 = \begin{pmatrix}-i & 0\\ 0 & i\end{pmatrix}$$

matrices obeying the quaternion algebra, τ_0 being the identity and the remaining matrices satisfying $\tau_i\tau_j = \epsilon_{ijk}\tau_k$. The operators γ_d, γ_c act on matrix indices and are given by

$$Q\gamma_d Q = 2\pi T \sum_{n_1n_2n_3n_4} Q_{n_1in_2i}q_{n_3in_4i}\delta_{n_1+n_3,n_2+n_4}$$
$$Q\gamma_c Q = 2\pi T \sum_{n_1n_2n_3n_4} Q_{n_1in_2i}Q_{n_3in_4i}\delta_{n_2+n_3,n_2+n_4} \tag{4}$$

The functional integral in (1) is restricted to hermitian matrices obeying

$$Q^+ = Q \qquad Q^2 = I \qquad tr\, Q = 0 \tag{5}$$

The first condition in (5) implies a $Q^a_{ni,mj} = Q^a_{mj,ni}$, $a = 0, 1, 2$ and $Q^3_{ni,mj} = -Q^3_{mj,ni}$, while because of the charge conjugation symmetry Q^i are real matrices.[6] These matrices are represented as a unitary transformation $Q = U^+\Lambda U$ of the saddle point matrix $\Lambda_{ni,mj} = \delta_{nm}\,\text{sign}(n)\delta_{ij}$ and describe the soft modes of the problem. They can be parametrized in terms of *unconstrained* quaternion[12] matrices V connecting positive and negative frequencies only.

$$Q = \begin{bmatrix}\sqrt{I - VV^+} & V\\ V^+ & -\sqrt{I - V^+V}\end{bmatrix} \tag{6a}$$

$$V_{ni,mj} = V^0_{ni,mj}\tau_0 + i\tau_1 V^1_{ni,mj} + i\tau_2 V^2_{ni,mj} + \tau_3 V^3_{ni,mj} \tag{6b}$$

Using this parametrization it is easy to verify that the constraints (5) are fully satisfied, while the quaternion structure (6b) is preserved in (3) because the productg (and any algebraic operation as in 6a) of two quaternions is again a quaternion of the same type (6b). Introducing the representation 6 in Eqs. 1–2 reduces the problem to functional integration over four real matrix fields $V^a_{ni,mj}$, $a = 0, 1, 2, 3$ which can be treated by standard perturbation theory. The parameterization of Eqs. (6) has the additional advantage of allowing the calculation of the renormalization of the one particle density of states which has not been calculated before in the spin orbit case. To carry out the perturbation theory we separate the quadratic part

$$L_d = \frac{1}{2t}\sum_{i=0}^{3}\int \nabla V^i_{nm}\nabla V^i_{nm} + z(\epsilon_n - \epsilon_m)V^i_{nm}V^i_{nm}$$

$$L_{ee} = \frac{2\pi t\Gamma_s}{t}\sum_{n_1n_2n_3n_4}\delta_{n_1+n_3,n_2+n_4}\sum_{i=0,3}V^i_{n_1n_2}V^i_{n_4n_3} \tag{7}$$

$$-\frac{2\pi t\Gamma_c}{t}\sum_{n_1n_2n_3n_4}\delta_{n_1+n_2,n_3+n_4}\sum_{i=1,2}V^i_{n_1n_2}V^i_{n_4n_3}$$

which defines the propagators of the theory.

$$\langle V^i_{n_1n_2}V^i_{n_4n_3}\rangle = tL(q,\omega)\delta_{n_1n_2}\delta_{n_2n_2} + 4\pi Tt\Gamma_s(q,\omega)L^2(q,\omega)\delta_{n_1+n_3,n_2+n_4} \qquad i = 0, 3 \tag{8}$$

$$\langle V^i_{n_1n_2}V^i_{n_4n_3}\rangle = tL(q,\omega)\delta_{n_1n_4}\delta_{n_2n_3} - 4\pi Tt\Gamma_c(q,\omega)L^2(q,\omega)\delta_{n_1+n_2,n_3+n_4} \qquad i = 1, 2 \tag{9}$$

with

$$L(q,\omega) = \frac{1}{(q^2 + z\omega)}\quad \Gamma_s(q,\omega) = \frac{\Gamma_s(q^2 + z\omega)}{q^2 + (z - 2\Gamma_s)\omega}$$

and

$$\Gamma_c(q,\omega) = \frac{\Gamma_c}{1 + \frac{\Gamma_c}{z}\ln\frac{\Lambda}{q^2 + z\omega}} \quad \textit{with}\quad \epsilon_{n_4}\epsilon_{n_1} > 0 \qquad \epsilon_{n_3}\epsilon_{n_2} < 0 \tag{10}$$

$\omega = \epsilon_{n_1} - \epsilon_{n_2}$, Λ is an ultraviolet cutoff. The part of the propagator which is diagonal in the energy indices is denoted by a wavy line in Fig. 1a while the non-diagonal part of the propagator is represented in Fig. 1b, the dot denotes an amplitude Γ_s or Γ_c.

The interaction vertices between the diffusion modes are obtained by inserting the parameterization (6) for the order parameter in the Lagrangian of Eq. (1) and collecting the terms which are cubic and quartic in the field V which are needed for a one loop calculation. The interaction vertices are shown in Fig. 1. Diagram 1c is generated by expansion of L_d while the terms generated by expansion of L_{ee} are represented by diagram 1b–1h. The notation here follows

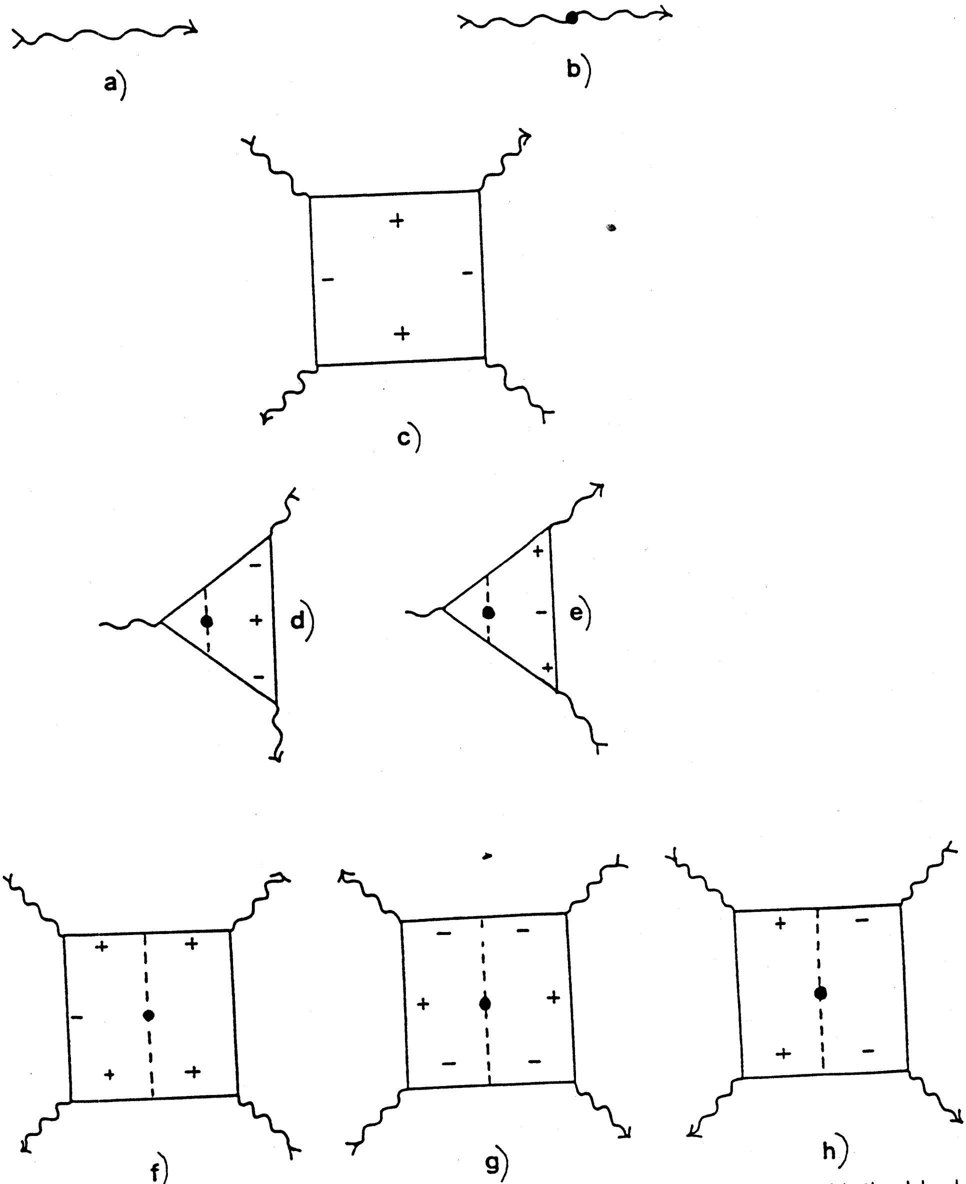

FIGURE 1 Graphical representation of the vertices. The dashed line with the black dot stands for a dynamical amplitude with the related energy and spin conservation laws. The incoming and outcoming arrows indicate formally the energy spins of the sides in a given vertex. For simplicity these signs are explicitly drawn.

Ref. (11), the wavy lines represent the quaternion field V. The Γ_s 4-legged vertices have V^0 and V^3, V^2 and V^1 or V^i and $V^i i = 0\ 1\ 2\ 3$ lines on opposite sides of the amplitude dashed line. While the Γ_c 4-legged vertices have V^3 and V^1, V^2 and V^0, V^3 and V^2 fields attached to the right or the left of the amplitude dashed line.

The one particle density of states is evaluated[2] from the expectation value of the matrix Q^0_{nn}. Using Eq. (6) we find to one loop order:

$$\langle Q^0_{nn} \rangle = 1 - \frac{1}{2} \sum_i \langle (V^i V^{i+})_{nn} \rangle \tag{11}$$

The corresponding diagrams are shown in Fig. (2). The line denotes an insertion of $\sum_{mi} V^i_{nm}(q) V^i_{nm}(-q)$ the $i = 0, 3$ propagators are contracted with a singlet amplitude while the $i = 1, 2$ propagators are contracted with a Cooper ladder. Requiring $\xi^{-1} \langle Q^0_{nn} \rangle$ to be nonsingular we find for, $\xi = \frac{N}{N_o}$, which is the ratio of the bare to the renormalized density of states:

$$\xi = 1 - \frac{t}{2} \int \frac{d^d q}{\pi^{d/2}} \int d\omega [2\Gamma_s(\omega, q) - 2\Gamma_c(\omega, q)] L^2(q, \omega) \tag{12}$$

We work to lowest order to Γ_c and perform the frequency integrations which are, to this order, independent of the ultraviolet cutoff. The remaining momentum integral is infrared divergent and is taken between the upper cutoff Λ and the lower momentum cutoff $\lambda\Lambda$ ($\lambda\Lambda < q < \Lambda$).

$$\xi = 1 + \frac{t\Lambda_c}{z} \ln \frac{1}{\lambda^2} - \frac{t}{\epsilon} \left[\left(1 - \frac{2\Gamma_s}{z} \right)^{\epsilon/2} - 1 \right] \ln \frac{1}{\lambda^2} \tag{13}$$

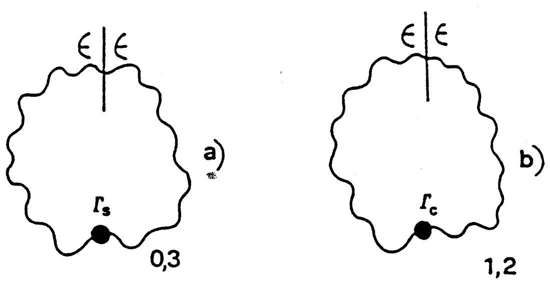

FIGURE 2 DOS diagrams at one loop order. The slashes mean that either the upper or the lower frequency of the two separated propagators has to be fixed. The numbers written near the dynamical amplitudes stands for the possible quaternion components to be considered in the contractions.

contracting the vertices in Fig. 1b with the propagators in Fig. 1a we obtain the one loop renormalization of the propagator (see Fig. 2).

The diagrams which are diagonal in the energy indices, termed self energy diagrams, are shown in Fig. 3. The inverse propagator of the theory which is diagonal in the energy indices is expected to scale as $L^{-1}(\omega = 0) = \xi^{-2}[Dk^2]$. Evaluating the diagrams in Fig. 3 we find

$$\delta(\xi^{-2}Dk^2) = \frac{k^2}{2}\int \frac{dq^d}{\pi^{d/2}} L(q, 0)$$

$$- \int d\omega \frac{dq^d}{\pi^{d/2}} [L(q + k, \omega) - (k^2L^2(q, \omega) + L(q, \omega))][\Gamma_s(q, \omega) - \Gamma_c(q, \omega)] \quad (14)$$

combining Eqs. (13) and (14) one finds the correction to the diffusion constant

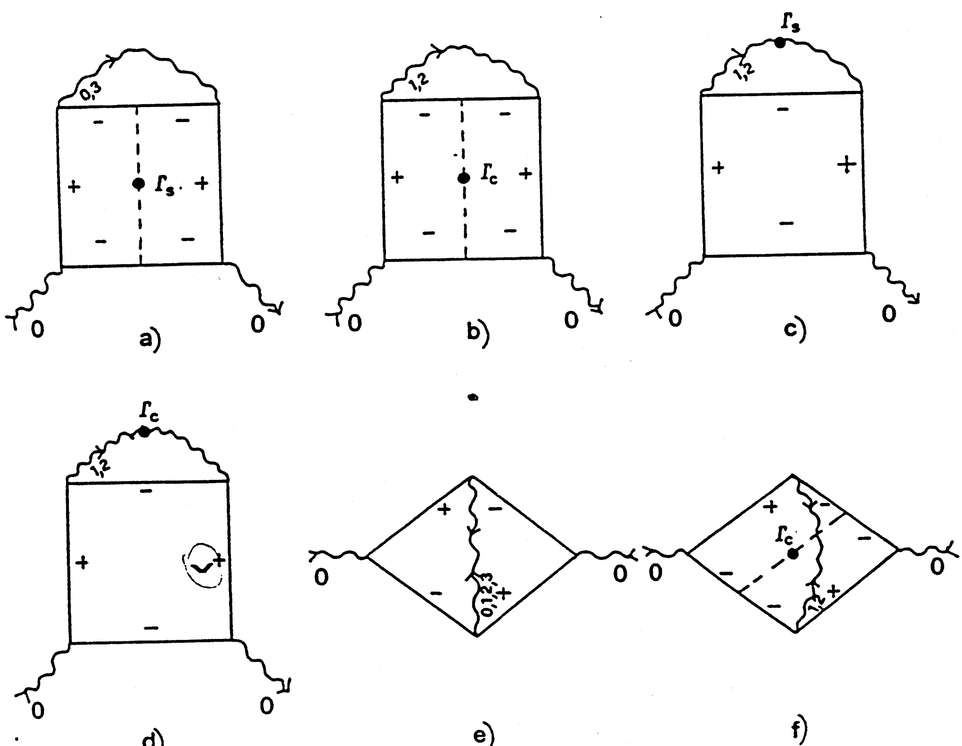

FIGURE 3 One loop order self energy diagrams. Here the notation are the same as in Fig. 2. In such diagrams the upper and lower energy indices are independently conserved in the external legs. The external quaternion indices are fixed to the identity component.

$$\delta D = \left\{\frac{1}{2} - \left[1 + \frac{1 - \frac{2\Gamma_s}{z}}{\frac{2\Gamma_s}{z}} \ln\left(1 - \frac{2\Gamma_s}{z}\right)\right] - \frac{\nu\Gamma_c}{z}\right\} \ln \lambda^{-2} \tag{15}$$

where λ is again the ratio of the upper and lower momentum cutoffs in the integral.

We now consider the correction to the propagators $\langle V_i V_i \rangle$ which are off diagonal in the energy indices, for $i = 0, 3$ and $i = 1, 2$ those are the corrections to the singlet and Cooper amplitude respectively. The diagrams which renormalize the Cooper amplitude are shown in Fig. 4. Diagrams (c), (d) and (e) add up to

$$\delta(\xi^{-2}\Gamma_c)_{c-e} = -2t \int dg\Gamma_s(\omega, q)L^2(q, \omega) \tag{16}$$

This term is the renormalization of the density of states due to the singlet part of the interaction. Notice that there are no Γ_c^2 diagrams and the Cooper contribution to the one particle density of states is not canceled. (a) and (b) add up to

$$\delta(\xi^{-2}\Gamma_c)_{a-b} = t \int d^dq\Gamma_s(q, 0)L(q, 0) = t\Gamma_s \ln \lambda^{-2} \tag{17}$$

combining Eqs. 16–17 with Eq. 12 we find $\delta\Gamma_c = \delta(\xi^{-2}\Gamma_e) + 2\delta(\xi)\Gamma_c$, that is

$$\delta\Gamma_c = t\left(2\frac{\Gamma_c^2}{z} + \Gamma_s\right) \ln \frac{1}{\lambda^2} \tag{18}$$

To that one has to add the usual logarithmic contribution coming from the Cooper channel summation, see diagram (f), arriving at

$$\delta\Gamma_c = t\left(2\frac{\Gamma_c^2}{z} + \Gamma_s\right) \ln \frac{1}{\lambda^2} - \frac{\Gamma_c^2}{z} \ln \frac{1}{\lambda^2} \tag{19}$$

Finally, we turn to the corrections to the singlet amplitude. In this case the diagrams coming from the contraction of the three point vertices (5a, 5b and 5c) exactly cancel the DOS contribution (both the singlet and the Cooper corrections) to the renormalized coupling Γ_s. The remaining diagrams (5d and 5e) directly provide a correction to the singlet amplitude after the mentioned cancellation of the DOS contribution:

$$\delta\Gamma_s = -\frac{t}{2} \int \frac{dq^d}{\pi^{d/2}} L(q^2, 0)(\nu\Gamma_s - \nu\Gamma_c) \tag{25}$$

We also check that the coupling z renormalizes as $2\Gamma_s$. Combining Eqs. (15),

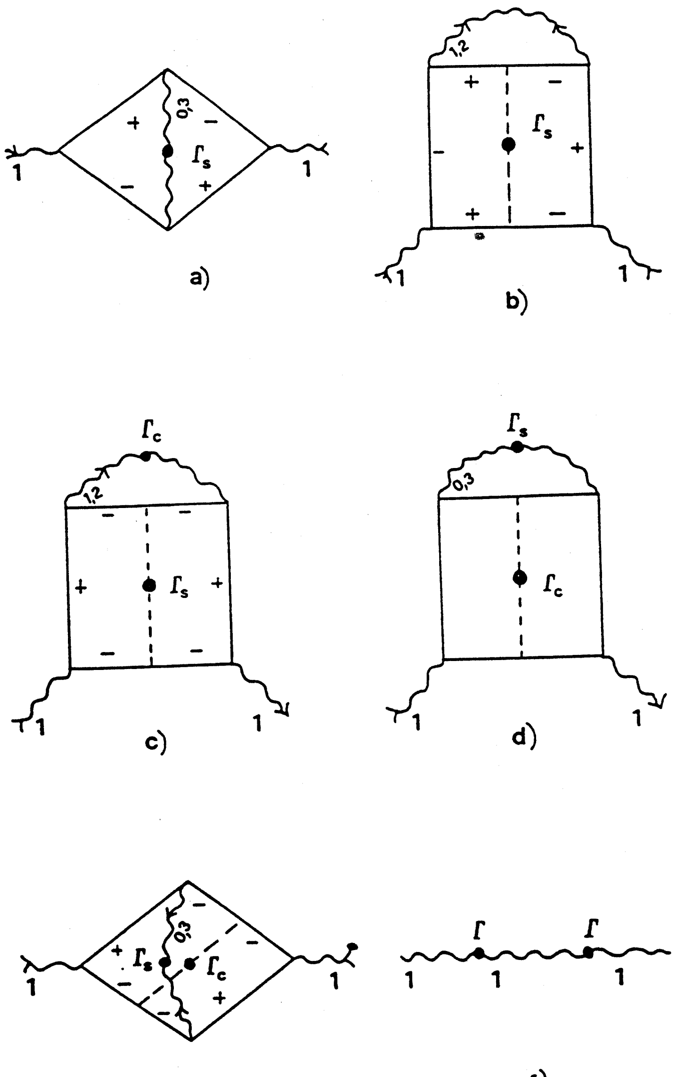

FIGURE 4 One loop order diagrams for the Cooper amplitude renormalization. Here we follow the same notation as in Fig. 2 while the external quaternion indices is fixed to the component τ_1. The energy conservation of the external legs must not be diagonal in the energy indices (this would be a self energy renormalization).

(19) and (25) one finds the R.G. equation for the couplings Γ_s, D, Γ_c of Castellani et al.[3] by taking the derivative of the recursion relations with respect to the logarithm of the inverse rescaling parameter $\ln(\lambda^{-1})$

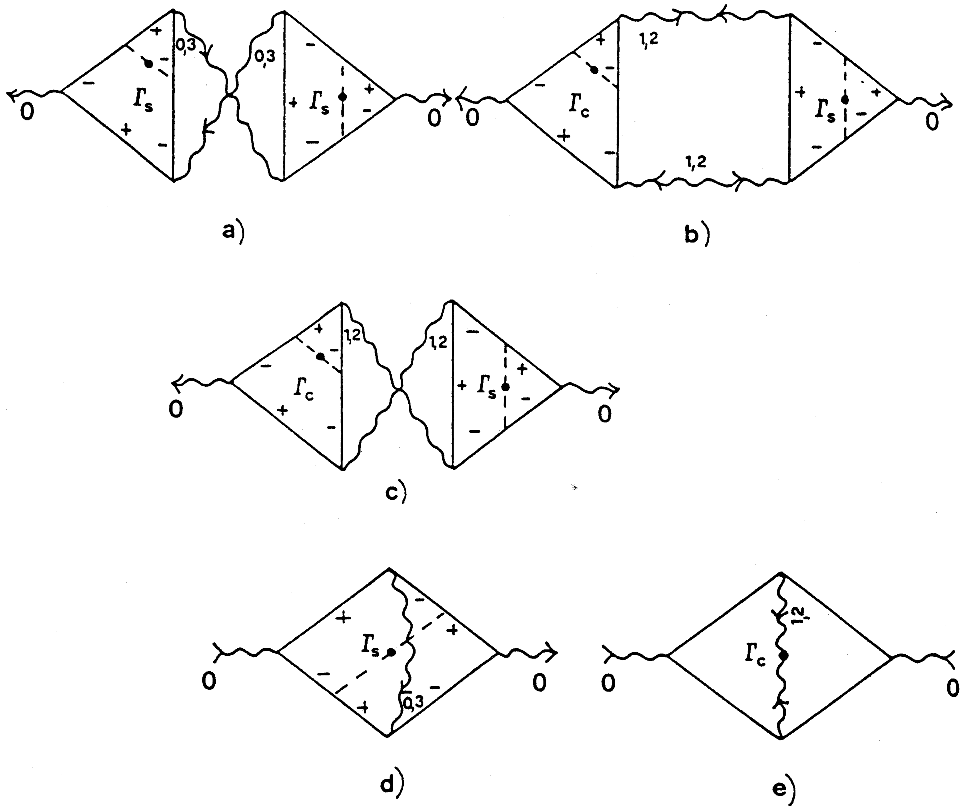

FIGURE 5 One loop order diagrams for the singlet amplitude renormalization. The notation are the same as in Fig. 2. The external quaternion indices are fixed to the identity component.

$$\dot{D} = \left\{ 1 - \left[2 + \frac{1 - \frac{2\Gamma_s}{z}}{\frac{\Gamma_s}{z}} \ln\left(1 - \frac{2\Gamma_s}{z}\right) \right] - \frac{2\Gamma_c}{z} \right\}$$

$$\dot{\Gamma}_s = -t[\Gamma_s - \Gamma_c] \qquad (27)$$

$$\dot{\Gamma}_c = t\left[\frac{4\Gamma_c^2}{z} + 2\Gamma_s\right] - \frac{2\Gamma_c^2}{z} \qquad (28)$$

II. THE CRITICAL EXPONENTS

We consider here the long range case where $2\Gamma_s = z$, a condition which is preserved by the R.G. eqs.

We rewrite Eqs. (26)–(28) in terms of only two variables $t = \frac{1}{D}$ and $\gamma = \frac{\Gamma_c}{z}$. In $2 + \epsilon$ dimensions the equations for t acquires an additional term from its bare dimensionality.

$$\dot{\gamma} = [-2\gamma^2 + t(1 + \gamma + 2\gamma^2)] = \beta_\gamma(\gamma, t) \tag{29}$$

$$\dot{t} = -\epsilon t + t^2(1 - 2\gamma) = \beta_t(\gamma, t) \tag{30}$$

Here we notice that these renormalization group equations which are valid to one loop order allows the determination of the critical indices to order $\epsilon^{3/2}$ since there is an unstable fixed point at

$$t^* = \epsilon(1 + \sqrt{2\epsilon}) \qquad \gamma^* = \sqrt{\frac{\epsilon}{2}} + \frac{3}{4}\epsilon \tag{31}$$

At this fixed point z scales to zero as $z(\lambda) \sim \gamma^{-x}$ as $\lambda \rightarrow 0$ with $x = -t^*(1 - 2\gamma^*) = -\epsilon + O(\epsilon^2)$; x appears in the specific heat exponent $C_V \propto T^{\frac{-x}{d+x}}$ at the transition. Linearizing around the fixed point we find the critical index $\lambda^{-1} \propto \xi_c \propto (t - t^*)^{-\nu} \propto (n - n_c)^{-\nu}$

$$\frac{1}{\nu} = \epsilon\left[1 - \sqrt{\frac{\epsilon}{2}}\right] \tag{32}$$

where ξ_c is the correlation length and n is the impurity concentration. Then the conductivity index is determined as

$$\sigma \sim (n - n_c)^s \qquad s = \epsilon\nu = 1 + \sqrt{\frac{\epsilon}{2}} \tag{33}$$

III. THE ONE PARTICLE DENSITY OF STATES

The density of states exponents are obtained following Ref. 2. We rewrite Eq. (13) as

$$\frac{\delta N}{N} = \left[\frac{t}{\epsilon}\left[\left(1 - \frac{2\Gamma_s}{z}\right)^{\epsilon/2} - 1\right] + \frac{t\Gamma_c}{z}\right] \ln\frac{1}{\lambda^2} \tag{34}$$

In the long range case $\frac{2\Gamma_s}{z} = 1$ and the logarithmic series has a coefficient of order $\frac{t^*}{\epsilon}$; as pointed out first by Finkel'stein.[2]

$$\frac{\delta N}{N} = -\left[\frac{t^*}{\epsilon} + o(\epsilon)\right] \ln\frac{1}{\lambda^2} \tag{35}$$

which at the critical point exponentiate to $N(\lambda) \propto \lambda^\theta$

$$\theta = 2 + 2\sqrt{2\epsilon} \tag{37}$$

The R.G. analysis suggests a scaling form for the one particle density of states[2]

$$N(T, \delta n) = \lambda^\theta N\left(\frac{T}{\lambda^{d+x}}, \delta n \lambda^{-\frac{1}{\nu}}\right) \qquad d + x = 2 + o(\epsilon^2) \tag{39}$$

and setting $\lambda = (\delta n)^\nu$ we find the exponent which characterize the vanishing of the density of states at the Fermi surface as one approaches to metal insulator transition,

$$N(0) \propto \delta n^{\nu\theta} \quad with \quad \nu\theta = \frac{2}{\epsilon}\left(1 + \frac{3}{\sqrt{2}}\epsilon^{1/2}\right) \tag{40}$$

On the other hand from Eq. 39 λ scales as $T^{\frac{1}{d+x}}$, from which we derive the vanishing of the one particle density of states at the transition $n = n_c$ as a function of temperature with an exponent $N(T) \propto T^\beta$:

$$\beta = \frac{\theta}{d + x} = 1 + \sqrt{2\epsilon} \tag{41}$$

CONCLUSIONS

Tunneling experiments were performed in NbSi films[10] close to the metal insulator transition. Conductivity measurements in the same films are consistent with $\sigma = (n - n_c)^s$ with $s \simeq 1$. The tunneling characteristics show metallic behaviour, i.e. $N(E) = N(0)\left[1 + \sqrt{\frac{E}{\Delta}}\right]$, for energies E less than a crossover energy scale Δ, Δ is seen to scale roughly as the second power of $(n - n_c)$. This agrees well with the results of the ϵ expansion because such exponent should be given, using Eq. (39), by $\nu(d + x) \simeq 2$. $N(0)$ is seen to scale as $(n - n_c)$, giving $\nu\theta = 1$. This exponent is very far from the result of Eq. (40). The fact that the ϵ expansion truncated to lowest order does not provide an accurate estimate of the critical exponents is not surprising since even for magnetic systems the $2 + \epsilon$ expansion around the lower critical dimension is far less accurate than the expansion around the upper critical dimension. The presence of fractional powers in the ϵ expansion of the critical exponents makes the nature of the series even more difficult to interpret, and we cannot extrapolate to the physical dimension $d = 3$, $\epsilon = 1$ with reasonable accuracy.

It is hard to extract from the data (10) the value of the exponent β. The value $\beta = \frac{1}{3}$ quoted in Ref. (10) is inconsistent with the scaling law $(d + x)\beta = \theta$ (see Eq. 41) which we derived from Eq. 39 and first obtained in Ref. 2. If the experimental value of β is indeed $\frac{1}{3}$ one would have to explain theoretically the violations of the simple scaling law. It is important to note that the ratio of the short range part of the charge compressibility (which is the compressibility of the electron gas and its neutralizing backround) to the specific heat, denoted by $\frac{z_1}{z}$ in Ref. 4 diverges at the transition. One could suspect violations of scaling if this physical quantity was represented by a dangerously irrelevant operators that entered the expression for the density of states, in which case the extraction of the exponent θ using Eq. (39) would be incorrect. However, we checked that the two loop order most divergent term (i.e. to second order in t) in the expansion of the density of states is consistent with the lowest order calculation giving a strong support to the procedure for extracting the critical index of the density of states originally suggested by Finkel'stein.[2]

Another possibility is the scaling as a function of temperature (which is calculated in the theory as in Eq. (41)) is different from the scaling of the density of states at criticality as a function of energy since a finite temperature and a finite distance from the fermi level correspond to moving away from criticality in different eigendirections of the renormalization group. This hypothesis would be confirmed if a careful measurement of the density of states at the Fermi surface at the metal-insulator transition as a function of temperature would give a critical index β different from $\frac{1}{3}$. More theoretical and experimental work is needed to elucidate this issue.

We obtained for the first time a nontrivial correction to the index s for the conductivity in the interacting case. It differs from the usual ones $\nu = \frac{1}{\epsilon}$, $\nu = \frac{1}{2\epsilon} - \frac{3}{4} + O(\epsilon)$ calculated in the noninteracting case.

Notice that to order $\epsilon^{3/2}$ the quasi particle diffusion constant[4] $\frac{D}{z}$ is unrenormalized. Since the equations for z and t are rather different, we don't expect this equality to be valid to all orders. A measurement of the thermal diffusion constant or a two loop calculation is therefore necessary to analyze the critical behaviour of the quasi-particle diffusion constant close to the metal insulator transition.

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