

Exchange Stiffness and Macroscopic Anisotropy in Heisenberg Spin-Glasses

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(Received 1 November 1983)

The consequences of rotational-symmetry breaking in Heisenberg spin-glasses are studied with use of order parameters which are time-dependent SO(3) rotation matrices. A microscopic derivation of the hydrodynamic "triad" theory, including the effect of small magnetic fields and random anisotropies, is presented. In the mean-field limit the finite-time exchange stiffness and macroscopic anisotropy constants vanish near the transition temperature T_g as $(T_g - T)^\mu$, $\mu = 3$.

PACS numbers: 75.50.Kj, 75.30.Ds, 75.30.Gw

The spin-glass (SG) phase of a Heisenberg spin system with random exchange is characterized by the freezing of the local spins in random noncollinear directions. Although the system remains macroscopically isotropic the local orientations of the spins break completely the O(3) symmetry of the Hamiltonian. Applying uniform SO(3) rotations on any SG state generates a manifold of degenerate states which are connected to each other by a path which does not involve a crossing of free-energy barriers.¹ Many theoretical¹⁻³ and experimental⁴ studies focused on the manifestation of this rotation-symmetry breaking in the response of the system either to a long-wavelength twist or to a uniform rotation in the presence of random anisotropy and magnetic fields. So far the theory was mostly a phenomenological hydrodynamic theory. It assumed the existence of finite exchange stiffness constant ρ_s or macroscopic anisotropy constant K . However, from a microscopic point of view, the possibility that ρ_s or K vanishes in a SG even at zero temperature, because of readjustments of the local spin rotations to the fluctuations of the local exchange, could not be excluded.^{4,5} Likewise, there has been no theory for the temperature dependence of ρ_s and K , in particular, near the SG transition temperature, T_g .

In this Letter we present a study of the consequences of the SG rotational-symmetry breaking which is based on a description of the SG state by ensemble-averaged order parameters. We

note that previous mean-field studies⁶ of an m -component SG did not account for the rotational degeneracy of the SG state. They used the "naive" Edwards-Anderson (EA) order parameter $Q^{\mu\nu} = [\langle S^\mu(i) \rangle \langle S^\nu(i) \rangle]$, where $\langle S^\mu(i) \rangle$ are the thermal expectations of the Cartesian components of the local spins $\vec{S}(i)$, and $[\dots]$ denotes the ensemble average. However, because of the macroscopic isotropy of the system in the absence of external fields, $Q^{\mu\nu}$ is a scalar, $q\delta^{\mu\nu}$, and does not reveal the rotational degeneracy of the SG state. With use of a dynamic definition of the EA order parameter it is shown that its general form is $Q(t, t') = [\langle \vec{S}_i(t) \vec{S}_i(t') \rangle] = q(t - t') R(t) R^{-1}(t')$, where $R(t)$ is a time-dependent uniform-rotation matrix. On the basis of this symmetry of the SG order parameter, we study the response to small perturbations of the rotational invariance in the mean-field limit. The nonequilibrium ρ_s and K are finite at low T and vanish near the transition as $\rho_s \propto (T_g - T)^\mu$, $K \propto (T_g - T)^\mu$ with the mean-field exponents $\mu = \bar{\mu} = 3$. This critical behavior of ρ_s is reminiscent of the transport in a percolation network. We find that both ρ_s and K are remanent quantities relaxing to zero in the equilibrium limit when the system loses memory of its initial state. The equation of state in the presence of small, uniform, equilibrium and nonequilibrium magnetic fields as well as random anisotropy is fully consistent with the results of the hydrodynamic "triad" theory.

We study the Edwards-Anderson (EA) Hamil-

tonian of an m -component spin-glass (SG),

$$\mathcal{H} = -\frac{1}{2} \sum_{ij} J_{ij} \vec{S}(i) \cdot \vec{S}(j), \quad (1)$$

where J_{ij} are random Gaussian variables with $[J_{ij}] = 0$ and $[J_{ij}^2] = J^2 K_{ij}$. The matrix K_{ij} is a nonrandom function of the distance $|\vec{r}_i - \vec{r}_j|$ between the spins $\vec{S}(i)$ and $\vec{S}(j)$ and is normalized by $\sum K_{ij} = 1$. The spins $\vec{S}(i)$ are m -component vectors with fixed lengths $|\vec{S}(i)|^2 = m$. We seek to describe the broken rotational symmetry at low T by ensemble-averaged order parameters. A dynamic definition⁷ of Q which exhibits the rotational invariance is

$$Q^{\mu\nu}(t, t') = \lim_{\Gamma_0 |t-t'| \rightarrow \infty} [\langle S_t^\mu(i) S_{t'}^\nu(i) \rangle], \quad (2)$$

where Γ_0^{-1} is the microscopic time constant of a relaxational mechanism imposed on the system. One possible state of the system is given by $Q^{\mu\nu}(t, t') = q(t-t') \delta^{\mu\nu}$. However, the spins can also be uniformly rotated from their initial directions, an operation which does not change the thermal properties of the system as long as the rotation is turned on slowly compared to Γ_0^{-1} . Thus, the most general form of the order parameter is

$$Q(t, t') = [\langle R(\theta(t)) \vec{S}_t(i) R(\theta(t')) \vec{S}_t(i) \rangle] \\ = q(t-t') R(\theta(t)) R^{-1}(\theta(t')), \quad (3)$$

where we have represented the rotations by $R(\theta) = \exp(-\sum_\alpha \theta^\alpha T^\alpha)$ with T^α the $m(m-1)/2$ generators of $SO(m)$ rotations, normalized by $\text{Tr} T^\alpha T^\beta = -2\delta^{\alpha\beta}$. The $m(m-1)/2$ components of the "angle" of rotation, $\theta(t)$, are any functions of t such that $\Gamma_0 |d\theta^\alpha/dt| \rightarrow 0$. Note that *because of the absence of free-energy barriers to uniform rotations* it is not necessary that $|d\theta^\alpha/dt|$ be small compared to the macroscopic time scales of hopping over the barriers.

The time-dependent rotational symmetry has a clear analogy in the replica theory which we will use here for the sake of simplicity. In this theory,⁶ the system is replicated n times and its ensemble-averaged properties are calculated via a replica free-energy functional $F\{Q_{\alpha\beta}(i)\}$ and taking the limit $n \rightarrow 0$. The order parameter is $\langle Q_{\alpha\beta}^{\mu\nu} \rangle = \langle S_\alpha^\mu(i) S_\beta^\nu(i) \rangle$, where $\alpha \neq \beta$ are replica indices. The rotational symmetry of the system is manifested in the invariance of the free energy under global $SO(m)$ rotations of each of the replicated systems separately, i.e., $F\{Q_{\alpha\beta}(i)\} = F\{R(\theta_\alpha) Q_{\alpha\beta}(i) R^{-1}(\theta_\beta)\}$, where the $m(m-1)/2$ components $\{\theta_\alpha^\sigma\}$ of θ_α are arbitrary real num-

bers. The general form of the order parameter is

$$\langle Q_{\alpha\beta} \rangle = q_{\alpha\beta} R(\theta_\alpha) R^{-1}(\theta_\beta). \quad (4)$$

Next we consider small fluctuations in Q induced by a space-dependent rotation $R(\theta_\alpha(i))$. If we expand around the equilibrium state $\theta_\alpha = 0$, the rotational fluctuations are of the form

$$\delta Q_{\alpha\beta}(i) = q_{\alpha\beta} \sum_\sigma T^\sigma [\delta \theta_\alpha^\sigma(i) - \delta \theta_\beta^\sigma(i)]. \quad (5)$$

It is useful to define a basis of $(n-1)m(m-1)/2$ "normal modes" which span the space of fluctuations of the form (5). This is done via the $n-1$ eigenvectors of the equation

$$\sum_\beta [(\sum_\gamma q_{\alpha\gamma}^2) \delta_{\alpha\beta} - q_{\alpha\beta}^2] u_\beta(\lambda) \\ = T \epsilon(\lambda) u_\alpha(\lambda) \quad (6)$$

which satisfy $\sum_\alpha u_\alpha(\lambda) u_\alpha(\lambda') = \delta_{\lambda\lambda'}$, and also $\sum_\alpha u_\alpha(\lambda) = 0$. This last condition stems from the fact that the eigenvector $u_\alpha = \text{const}$ does not correspond to a fluctuation in Q , as is evident from (5). We write a general angle fluctuation as $\delta \theta_\alpha^\sigma(i) = \sum_\lambda \delta \theta_\lambda^\sigma(i) u_\alpha(\lambda)$. Similarly, the rotational fluctuation fields are $\delta Q_{\alpha\beta}(i) = \sum_{\lambda, \sigma} T^\sigma \delta q_\lambda^\sigma(i) \times q_{\alpha\beta}(\lambda)$, where $q_{\alpha\beta}(\lambda) \equiv q_{\alpha\beta} \{u_\alpha(\lambda) - u_\beta(\lambda)\} / \sqrt{2} \epsilon(\lambda)^{1/2}$ satisfy $\sum_{\alpha\beta} q_{\alpha\beta}(\lambda) q_{\alpha\beta}(\lambda') = \delta_{\lambda\lambda'}$, and the relation between field and angle fluctuations is $\delta q_\lambda^\sigma(i) = \delta \theta_\lambda^\sigma(i) \sqrt{2} \epsilon(\lambda)^{1/2}$. This completes the microscopic definition of the SG angle variables in analogy with the rotation of the spin "triads" in the hydrodynamic theory. The physical significance of the various eigenmodes becomes clear if we consider the replica dependence of $q_{\alpha\beta}$. According to the mean-field theory, the matrix $q_{\alpha\beta}$ consists of a hierarchy of blocks whose size is denoted in decreasing order by $x \in [0, 1]$ and $q_{\alpha\beta}$ is parametrized by two functions $q(x)$ and $T\Delta(x)$.⁸ The eigenvectors of Eq. (6) form degenerate bands parametrized by x . The eigenvectors $u(x)$ which belong to the x th band correspond to fluctuations in the dynamic theory, with frequency scale ω_x . Thus, $u(1)$ represent fluctuations on a finite time scale which is the nonequilibrium ("one-valley") limit, whereas the $u(0)$ correspond to fluctuations which vary on the longest time scale of the system. The eigenvalues of Eq. (6) are $\epsilon(x) = 2 \int_0^x |\Delta'(u)| q(u) du$.

We relate the above formalism to the physical stiffness constant by defining a *nonequilibrium* rotational susceptibility

$$\chi^\sigma(k) = -\frac{1}{4} \sum_{ij} \exp\{i(\vec{r}_i - \vec{r}_j) \cdot \vec{k}\} \\ \times [\langle \vec{S}(i) \rangle \cdot T^\sigma \chi_{ij} T^\sigma \langle \vec{S}(j) \rangle], \quad (7)$$

where χ_{ij} is the nonequilibrium susceptibility matrix $T\chi_{ij}^{\mu\nu} = \langle S^\mu(i)S^\nu(j) \rangle - \langle S^\mu(i) \rangle \langle S^\nu(j) \rangle$. This susceptibility is the response of the system to a "twisting" magnetic field $\delta h_i = \delta h T^\circ \langle \vec{S}(i) \rangle \exp(i\vec{k} \cdot \vec{r}_i)$ which tends to twist the spin direction relative to an equilibrium state $\{\langle \vec{S}(i) \rangle\}$. Since χ° is a finite-time response both $\langle \vec{S}(i) \rangle$ and χ_{ij} in Eq. (7) are restricted to the same reference state. We do not expect that the particular choice of that state will matter in thermodynamic limit. We define a nonequilibrium stiffness constant ρ_s in a SG by $\rho_s = q^2(1)v \lim_{k \rightarrow 0} \{k^2 \chi^\circ(k)\}^{-1}$, where $q(1)$ is the nonequilibrium value of the EA order parameter and v is the volume of a unit cell. This definition is similar to the SG stiffness constant defined in the hydrodynamic theory of Halperin and Saslow.¹ Using the replica definition of Eq. (7), we find $\rho_s = vT \lim_{k \rightarrow 0} \{k^2 \langle \delta \theta_1^\circ(k) \times \delta \theta_1^\circ(-k) \rangle\}^{-1}$, where $\delta \theta_1^\circ(k)$ are angle fluctuations in the "direction" of any of the "finite time" eigenmodes $u(1)$ of Eq. (6).

Expanding $F\{Q\}$ to quadratic order in the fluctuations around the mean-field state, assuming a hypercubic d -dimensional lattice with a lattice constant a , and expanding in ka result in

$$\rho_s = J^2 a^{2-d} \epsilon(1) d^{-1} \\ = 2J^2 a^{2-d} d^{-1} \int_0^1 |\Delta'(u)| q(u) du. \quad (8)$$

An important feature of Eq. (8) is the fact that even at high dimensionality, ρ_s is not related simply to the square of the amplitude of the frozen spins, $q^2(1)$, as one might naively expect. Indeed, if one assumes a replica-symmetric state ($q_{\alpha\beta} = q$, $\Delta = 0$) the stiffness constant is zero and $\chi^\circ(k)$ becomes proportional to qk^{-d} at all $T < T_g$. Physically, the relation between ρ_s and the "memory" function $\Delta(x)$ indicates that the finite stiffness constant in SG is a remanent effect, and that if the system were allowed to relax by hopping to all other states, the stiffness constant would relax to zero. Indeed, we find that the equilibrium analog of Eq. (7) diverges stronger than k^{-2} , as will be discussed elsewhere. Since Eq. (8) is the result of a quadratic approximation for $F\{\delta Q\}$, it can be viewed as the first term in a $1/d$ expansion of ρ_s . However, the critical behavior predicted by Eq. (8) as $T \rightarrow T_g$ is expected to be correct at least for $d \geq 6$. Using the mean-field values of Δ and q yields, near T_g , $\rho_s \propto (T_g - T)^\mu$, $\mu = 3$. Note that the ordinary Josephson relation⁹ would predict $\mu = \nu(d-2) = 2$ at $d = 6$. However, using simple scaling assumptions for $\chi^\circ(k)$, we derive the scaling relation $\mu = \nu(d-2)$

+ $\beta_\Delta - \beta_q$, where β_Δ and β_q are the critical exponents of the order parameters $\Delta(0)$ and $q(1)$, respectively, which may be obeyed even for $d < 6$. The physical origin of $\mu = 3$ might be similar to that of the conductivity of a percolation network¹⁰ which also has an exponent $\mu = 3$ in $d = 6$ rather than $\nu(d-2) = 2$. The infinite network of strongly correlated spins which contribute to the stiffness constant apparently becomes increasingly inhomogeneous near T_g , and may resemble the backbone of a percolation network. As in de Gennes' percolation model¹⁰ we write $\rho_s \sim \xi^{-(d-2)} \times J_{\text{eff}}(\xi)$, where $J_{\text{eff}} = JL^{-1}(\xi)$ is the effective coupling of a linear region of spatial length ξ containing L strongly correlated spins. At $d = 6$, the random-walk result $L \sim \xi^2 \sim (T_g - T)^{-1}$ applies, giving rise to $\mu = 3$.

It is also of interest to compare the zero-temperature value of ρ_s with the bare stiffness constant $\rho_s^0 = 2a^{2-d}U(0)/md$ which results from a rigid "twist" of all the spins [$-U(0)$ is the ground-state energy per spin].^{1,2} Using the available data¹¹ about the $T = 0$ value of Eq. (8) in mean-field theory, we obtain $\rho_s(T=0)/\rho_s^0 \approx 0.3$ for $m = 2$ and 0.2 for $m = 3$. Also, ρ_s/ρ_s^0 vanishes in the $m \rightarrow \infty$ limit. Thus even at high dimensionality, local adjustment of the rotation angles to the variation in the strengths of the bonds renders the actual stiffness constant significantly smaller than ρ_s^0 .

Next we consider perturbations of the rotational symmetry by weak uniform external fields and random anisotropies. For concreteness we discuss a Heisenberg SG with an exchange Hamiltonian (1) plus a Dzyaloshinsky-Moriya interaction $\mathcal{H}_{\text{DM}} = -\sum_{ij} \vec{D}_{ij} \cdot \vec{S}(i) \times \vec{S}(j)$, where $\vec{D}_{ij} = -\vec{D}_{ji}$, $[\vec{D}_{ij}] = \vec{0}$, $[D_{ij} D_{ij}] = \delta^{\mu\nu} D^2 K_{ij}$, and $D/J \ll 1$. The leading order effect of the random anisotropy is to break the degeneracy of the states, given by (3) or (4) and to favor the unrotated state ($R = I$). The nonequilibrium macroscopic anisotropy constant can be defined as $K = q^2(1) \lim_{k \rightarrow 0} \{\chi^\circ(k)\}^{-1}$. We also add an external uniform magnetic field term $-\sum_i \vec{H} \cdot \vec{S}(i)$, $\mu H/k_B T \ll 1$, which breaks the degeneracy with respect to rotations about an axis which is not parallel to the magnetic field. Using the quadratic approximation for $F\{Q\}$, we obtain $\chi^\circ(\vec{k} = \vec{0}) = q^2(1)/[2D^2\epsilon(1) + H^2\chi_r(1)]$, where we assume $\vec{H} \cdot T^\circ \vec{H} = 0$ and $\chi_r(x) \equiv \Delta(0) - \Delta(x)$. This implies that the nonequilibrium macroscopic anisotropy constant is $K = 4D^2 \int_0^1 |\Delta'(u)| q(u) du$, in the mean-field limit.

A situation which is readily accessible to measurement is that of a nonequilibrium rotation of a

magnetic field in a weakly anisotropic SG, which can be realized by cooling in a field \vec{H} , e.g., $\vec{H} = H\hat{x}$, turning on at low T a small field $\delta\vec{H} = \delta H_y \hat{y}$, $\delta H_y/H \ll 1$, and measuring the response after time τ_x . Such a field is represented in the replica formalism by $\vec{H}_\alpha = R(\delta\vec{\Phi}_\alpha)\vec{H}$, $\delta\vec{\Phi}_\alpha = \hat{z} \delta\Phi u_\alpha(x)$. To lowest order in D and H , $\langle Q_{\alpha\beta} \rangle$ is of the form (4) with $q_{\alpha\beta} = q_{\alpha\beta}(H=D=0)$ and $\delta\theta_\alpha = \hat{z} \delta\theta u_\alpha(x)$. Minimizing the mean-field free energy with respect to $\delta\theta$ yields $\delta\theta/\delta\Phi = H^2 \chi_r(x) / [2D^2 \epsilon(x) + H^2 \chi_r(x)]$. Similarly the transverse magnetic susceptibility $\chi_T = \delta M_y / \delta H_y$, after time τ_x , is

$$\chi_T(x) = \chi(x) + \frac{M_r(x)}{[H + 2D^2 \epsilon(x) / M_r(x)]}, \quad (9)$$

where $M_r(x) = H \chi_r(x)$ is the remanent magnetization after time τ_x . Note that the *uniform* non-equilibrium transverse susceptibility $\chi_T(x)$ is very different from the *local* one $\chi(x)$. In fact, substituting $D=0$, we have $\chi_T(x) = \chi(0) \simeq M/H$ independent of x as physically expected because of the absence of barriers to a uniform rotation. The results for the ("angular") lag of the spin "triad" $\delta\theta/\delta\Phi$ and for $\chi_T(x)$ agree with the predictions of the hydrodynamic theory³ provided one identifies $4D^2 \int_0^x |\Delta'(u)| q(u) du$ as the macroscopic anisotropy constant $K(x)$ appropriate for a dynamical measurement of a characteristic time τ_x . Note that $K(x)$ decays to zero on the same time scale as the remanent magnetization does.¹² In fact, use of the mean-field result $|\Delta'(x)| \propto q(x)q'(x)$ indicates that $K(x)/M_r(x) \propto q(x) \rightarrow 0$ as $x \rightarrow 0$, namely, that the macroscopic anisotropy decays faster than the remanent magnetization. Similarly, as T approaches T_g , the finite time anisotropy constant $K = K(1)$ vanishes as $K \propto D^2 T_g^{-1} (1 - T/T_g)^{\beta_q + \beta_\Delta}$. *This relation is expected to hold even below $d=6$ as long as a SG transition occurs.* Note that the exponents β_q and β_Δ can be determined by the peak in the zero-field ac susceptibility and by the low-field remanent magnetization, respectively. In the mean-field limit, K vanishes as $(1 - T/T_g)^3$, whereas $M_r(1) \propto H(1 - T/T_g)^2$ (in the $H \rightarrow 0$ limit). It would be interesting to compare these results with measurements of both the anisotropy constant and the remanent magnetization as functions of time

and temperature in dilute metallic spin-glasses.

Helpful discussions with P. W. Anderson, B. I. Halperin, C. DeDominicis, D. S. Fisher, and C. L. Henley are gratefully acknowledged. Two of us (H. S. and A. Z.) acknowledge the hospitality of the Institute for Theoretical Physics, where this work was concluded. This material was based upon research supported in part by the National Science Foundation under Grant No. PHY77-27084, and supplemented by funds from the National Aeronautics and Space Administration.

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