Kinetic equation for strongly disordered systems. II. Interacting electrons

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Electronic transport, in the presence of strongly disordered impurities and including the electron-electron interaction, is described via a kinetic equation of the diffusive form for the energy distribution function of quasiparticles. This equation is obtained by combining the diagrammatic treatment of the linear-response functions with a suitable identification of the distribution function itself. All known results for the number, spin, and heat correlation functions are shown to be recovered from this unified approach, with the same spirit underlying the conventional Landau theory of Fermi liquids.

I. INTRODUCTION

The theory of electronic transport, in the presence of strongly disordered impurities and including the electron-electron interaction, has mainly focused on calculating the hydrodynamic form of the number,[1,2] spin,[3,4] and heat[5] density correlation functions, whereby the respective diffusion coefficients exhibit logarithmic singularities near two dimensions. The description of transport for interacting Fermi systems in the clean[6] or slightly dirty[7] limit has, on the other hand, relied on the Landau approach, which introduces at the outset a phase-space distribution function for quasiparticles and determines the correlation functions from its evolution via a kinetic equation.

This situation bears some analogies with the case of noninteracting electrons that has been considered in a previous paper[8] (henceforth referred to as paper I). In particular, one may wonder whether a meaningful distribution function could be defined in the dirty limit for the interacting case and whether an associated kinetic equation could be derived, in terms of which one may cast the results from perturbative calculations in a physically appealing form. This task is especially needed in view of the fact that Fermi-liquid concepts have been of common use in the theory of dirty systems, as stressed in Ref. 9. Moreover, the notion of quasiparticles in disordered media has already been introduced in Ref. 10, where it has been utilized to provide a physical picture of the metal-insulator transition in terms of localization of quasiparticles. The effective Landau picture resulting from previous work gives one confidence in obtaining a kinetic equation of the Landau type for a suitable distribution function of quasiparticles, as anticipated in Ref. 5.

The mismatch between the early use of Fermi-liquid concepts in very dirty metals and the lack of an associated kinetic equation was already pointed out in a paper by McMillan,[11] who approached the problem from a phenomenological point of view and introduced a semiclassical distribution function for quasiparticles labeled by their energy ε in isolation to describe the statistical behavior of the system. It had appeared, in fact, intuitively evident that, in the presence of strong disorder, the lack of momentum conservation would hinder the classification of the quasiparticle states in terms of a wave vector $\mathbf{k}$ as well as the identification of suitable $\mathbf{k}$-dependent Landau f functions. The phenomenological kinetic equation by McMillan, however, was not fully correct. It was later resumed and correctly modified in Ref. 5, where it has been stated to have a diffusive form of the same kind discussed in paper I for the noninteracting case. No derivation of the kinetic equation was, however, provided in Ref. 5.

In this paper we derive a transport equation for quasiparticles in very dirty systems relying on the same diagrammatic structure that has been extensively used for the correlation functions.[1-5] We will show that the combined effects of anelastic scattering and strong disorder reduce the energy of quasiparticles in isolation to being the sole relevant variable for the quasiparticle distribution function. We will find that the diffusion equation for the energy distribution function of quasiparticles in the form stated in Ref. 5 is actually correct, and that this equation is sufficient to recover all known results for the hydrodynamic correlation functions.

We refer the reader to paper I for a discussion on the procedure that is required to define a suitable distribution function in the presence of strong disorder and with the neglect of the electron-electron interaction. It was argued in paper I that the familiar Wigner distribution function fails to satisfy a kinetic equation in the presence of strong disorder because of a broadening that affects the otherwise sharp drop of the unperturbed distribution function at the Fermi surface. It was also remarked in paper I that alternative methods (such as the Baym-Kadanoff or Keldysh methods) to derive a kinetic equation in the presence of strong disorder appear considerably more involved from the computational point of view than the ordinary diagrammatic theory which is usually adopted to treat localization problems. In the present paper we shall assume that these arguments can be taken over *a fortiori* to the interacting case. We shall thus concentrate our efforts on assessing how the combined effects of interaction and strong disorder affect the description of transport via a kinetic equation, by relying on the methods described in paper I.

The plan of the paper is as follows. For pedagogical purposes in Sec. II we show how the ordinary Landau-Boltzmann equation can be derived in the limit of weak disorder, by suitably handling the diagrammatic structure adopted for the study of the metal-insulator transition for disordered electron systems. This derivation aims both at a tentative generalization of the Landau-Boltzmann equation for a **k**-dependent distribution function in the case of strong disorder, which is discussed in the Appendix, and at the extension to strong disorder of the diffusion equation for the energy distribution function. This main accomplishment of the paper is described in Sec. III. In Sec. IV we indicate how the diffusion equation for the energy distribution function reproduces all known results of the renormalized Fermi-liquid picture of strongly disordered systems. Section V gives our conclusions.

II. LANDAU-BOLTZMANN EQUATION WITH ISOTROPIC QUASIPARTICLE SCATTERING AMPLITUDES IN THE PRESENCE OF WEAK DISORDER

In this section we briefly review the derivation of the Landau-Boltzmann equation for quasiparticles in the presence of dilute impurities,[7] by relying on the same treatment of the diagrammatic structure that is adopted in the theory of the metal-insulator transition.[1-5] The method of this section will serve as a guide for extending the kinetic equation in the presence of strong disorder, as discussed in Sec. III and in the Appendix.

As in the noninteracting case treated in paper I, we limit ourselves to considering the linear response of the system to the action of an external scalar potential. (Magnetic and thermal couplings will be briefly considered in Sec. IV.) Recall that in the presence of interaction this limitation is actually a necessity since the quasiparticles are well-defined excitations of the system only in the neighborhood of the Fermi surface. In addition, the disorder is conveniently handled within the Gaussian approximation by assuming an on-site impurity potential $u(\mathbf{r})$ such that $\overline{u(\mathbf{r})u(\mathbf{r}')}=[2\pi N_0(\varepsilon_F)\tau]^{-1}\delta(\mathbf{r}-\mathbf{r}')$. Here, the overbar denotes the average over the impurity configurations, $N_0(\varepsilon_F)$ is the free-particle density of states at the Fermi level per spin component, and τ is the impurity collision time (for simplicity, throughout this paper we shall assume the presence of nonmagnetic impurities only). The strength of disorder is classified as usual in terms of the inverse dimensionless conductance $t=[2\pi N_0(\varepsilon_F)D_0]^{-1}$ where D_0 is the Drude diffusion coefficient. (We set $\hbar=1$ throughout.) In particular, we limit ourselves in this section to considering effects of order t^0, while in the next section we shall explicitly take into account effects of order t^1.

The interparticle interaction can be handled at order t^0 in the conventional way of the Landau theory of Fermi liquids, by performing whenever possible partial summations of diagrams to define the bare (i.e., not yet dressed by disorder) interparticle scattering amplitudes.[6,12] We then adopt the same simplification that has extensively been used in the theory of the metal-insulator transition,[1-5] and consistently neglect the angular dependence of the interparticle scattering amplitudes. (For the case of weak disorder considered here, the neglecting of the angular dependence of the scattering amplitudes reduces the effective mass of a quasiparticle to the bare mass of the corresponding particle.[6,12]) For convenience, we shall also neglect to indicate explicitly the single-particle (bare) renormalization constant a ($a=1$), although this does not imply that we disregard the incoherent part of the single-particle Green's functions. This part will, in fact, be absorbed in the (nonsingular) static scattering amplitude Γ_s^∞ introduced below.

The interparticle potential is regarded to be of short range. Otherwise, the direct Coulomb term of the scattering amplitude (leading to the self-consistent Hartree potential) could be isolated at the outset from the diagrammatic structure and its effect added directly to the external potential via the Silin procedure.[13,6,7]

We begin our derivation by considering the dynamical part δg of the distribution function introduced in paper I which represents the deviation from local equilibrium and is proportional to the external frequency Ω. [Notice that the normalization factor in Eq. (2.13) of paper I which defines δg should read 4π instead of 2π]. δg is obtained by solving the Bethe-Salpeter equation for the linear variation of the time-ordered single-particle Green's function in the presence of the driving field. To handle this equation, we introduce the quantity Γ_s^∞ of the ordinary Fermi-liquid theory,[6,12] by performing at the outset partial summations of diagrams in which all pairs of single-particle lines (with respect to which the sequences of diagrams of the Bethe-Salpeter equation are reducible) contain a large internal frequency with respect to Ω.[1] The above procedure amounts to taking $\Omega=0$ in the argument of the partial summations. (We consistently use in this paper the same notation as in paper I. Ω and $\mathbf{Q}$ thus stand for the external frequency and wave vector, respectively, and the subscript s identifies the singlet component of the scattering amplitude Γ which couples to the external scalar field. The label ∞ attached to Γ_s further signifies that the ratio $\mathbf{Q}/\Omega$ is taken to diverge.)

In this way, we construct the sequence of diagrams depicted in Fig. 1, where a disorder direct ladder can be inserted within each pair of $+-$ (retarded-advanced) single-particle lines. This sequence terminates *by construction* on the left side with a pair of $+-$ lines; on the right side, on the other hand, a final vertex Λ_s^∞ can be added which does not allow for any further insertion of disorder ladders. Notice that allowance for this terminal vertex insertion amounts to a renormalization of the

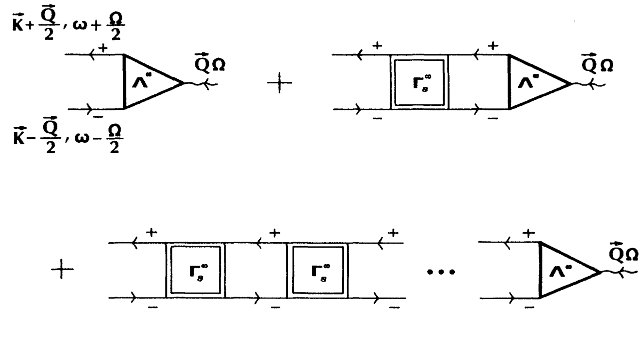

FIG. 1. Sequence of diagrams yielding the dynamical part of the distribution function $\delta g(\hat{\mathbf{k}},\omega;\mathbf{Q},\Omega)$ to linear order in the external scalar field. The labels $(+)$ and $(-)$ refer to the sign of the frequency argument of the single-particle Green's functions, with respect to the chemical potential. For clarity, the left entries of momentum and frequency are indicated for the first term only. Recall that the disorder ladders can be inserted only within a pair of $(+)$ and $(-)$ lines $(|\omega|<\Omega/2)$, thus yielding the frequency dependence of the hydrodynamic regime.

external field $\Phi_{\rm ext}\rightarrow\Lambda_s^\infty\Phi_{\rm ext}$. The vertex Λ_s^∞ satisfies the relation

$$\Lambda_s^\infty=1-iN_0(\varepsilon_F)\Gamma_s^\infty \ . \tag{2.1}$$

Γ_s^∞ is, in turn, related to its counterpart Γ_s^0 in the reverse limit of vanishing ratio $\mathbf{Q}/\Omega$ by the relation

$$\Gamma_s^0=\Gamma_s^\infty+\Gamma_s^\infty iN_0(\varepsilon_F)\Gamma_s^0 \ . \tag{2.2}$$

[Notice that, in the spirit of the conventional treatment in the theory of localization,[1-5] Eqs. (2.1) and (2.2) have been handled as if they were algebraic instead of integral equations.] Equation (2.2) can then be solved for Γ_s^∞ and the result inserted into Eq. (2.1), to give

$$\Lambda_s^\infty=\frac{1}{1+F_0^s} \ , \tag{2.3}$$

where the real quantity

$$F_0^s=iN_0(\varepsilon_F)\Gamma_s^0 \tag{2.4}$$

is the isotropic (and singlet) component of the Landau interaction function which enters the "effective potential" felt by the quasiparticles.

The sequence of scattering processes depicted in Fig. 1, once resummed with the further insertion of disorder direct ladders within the $+-$ single-particle lines, specifies the dynamical part δg of the distribution function in terms of the following integral equation:

$$(-i\Omega+iv_F\hat{\mathbf{k}}\cdot\mathbf{Q})\delta g(\hat{\mathbf{k}},\omega;\mathbf{Q},\Omega)+2i\Omega\delta(\omega)\left[\Lambda_s^\infty\Phi_{\rm ext}(\mathbf{Q},\Omega)+\frac{i}{2}N_0(\varepsilon_F)\Gamma_s^\infty\int_{-\infty}^{+\infty}d\omega'\int\frac{d\hat{\mathbf{k}}'}{S_d}\delta g(\hat{\mathbf{k}}',\omega';\mathbf{Q},\Omega)\right]$$
$$=-\frac{1}{\tau}\left[\delta g(\hat{\mathbf{k}},\omega;\mathbf{Q},\Omega)-\int\frac{d\hat{\mathbf{k}}'}{S_d}\delta g(\hat{\mathbf{k}}',\omega;\mathbf{Q},\Omega)\right] , \tag{2.5}$$

where v_F is the Fermi velocity and S_d stands for the solid angle in d dimensions. Notice that, as in the noninteracting case, the factor $\Omega\delta(\omega)$ on the left-hand side of Eq. (2.5) originates from the restriction $-\Omega/2<\omega<\Omega/2$ which distinguishes the dynamical contribution.

It is instructive to compare Eq. (2.5) with its counterpart in the absence of interaction at the same order t^0 we are considering here. At this order, the effect of the interaction is to replace the external field in the driving term by the "effective field" given by the expression within large parentheses on the left-hand side of Eq. (2.5). This remark suggests that in the presence of weak disorder the full (static plus dynamic) phase-space distribution function of quasiparticles δn can be obtained from the corresponding equation in the noninteracting case with the replacement of the external field by the above "effective field" [cf. Eq. (3.7) of paper I]. Upon setting

$$\delta g(\hat{\mathbf{k}},\omega;\mathbf{Q},\Omega)=2\delta(\omega)\gamma(\hat{\mathbf{k}};\mathbf{Q},\Omega) \ , \tag{2.6}$$

we can immediately perform the frequency integration in the effective field of Eq. (2.5) and write the two distinct first-order contributions to δn in the form

$$\delta n(\mathbf{k};\mathbf{Q},\Omega)=\delta g(\hat{\mathbf{k}},\varepsilon_{\mathbf{k}};\mathbf{Q},\Omega)-2\delta(\varepsilon_{\mathbf{k}})\left[\Lambda_s^\infty\Phi_{\rm ext}(\mathbf{Q},\Omega)+iN_0(\varepsilon_F)\Gamma_s^\infty\int\frac{d\hat{\mathbf{k}}'}{S_d}\gamma(\hat{\mathbf{k}}';\mathbf{Q},\Omega)\right] \tag{2.7}$$

with $\varepsilon_{\mathbf{k}}=\mathbf{k}^2/2m-\mu$ (μ being the chemical potential). Notice that the "effective field" in Eq. (2.7) can actually be expressed in terms of the full distribution function δn itself.[7] In fact, upon setting

$$\delta n(\mathbf{k};\mathbf{Q},\Omega)=2\delta(\varepsilon_{\mathbf{k}})\nu(\hat{\mathbf{k}};\mathbf{Q},\Omega) \ , \tag{2.8}$$

in analogy with Eq. (2.6), and recalling Eqs. (2.1) and (2.4), we arrive at the result

$$\Lambda_s^\infty\Phi_{\rm ext}(\mathbf{Q},\Omega)+iN_0(\varepsilon_F)\Gamma_s^\infty\int\frac{d\hat{\mathbf{k}}'}{S_d}\gamma(\hat{\mathbf{k}}';\mathbf{Q},\Omega)=\Phi_{\rm ext}(\mathbf{Q},\Omega)+F_0^s\int\frac{d\hat{\mathbf{k}}'}{S_d}\nu(\hat{\mathbf{k}}';\mathbf{Q},\Omega) \ . \tag{2.9}$$

The (Ward) identity (2.9) entails two basic assumptions of the Landau theory of Fermi liquids, namely, that a quasiparticle couples with the external field in the same way as a bare particle, and that besides this coupling the quasiparticles are acted upon by a self-consistent potential due to the other quasiparticles, which is linear in their distribution function δn.[6,12]

The Landau-Boltzmann equation in the presence of weak disorder can now be obtained by entering Eqs. (2.7) and (2.9) into Eq. (2.5). One recovers the well-known result[7] to linear order in the external field

$$(-i\Omega+iv_F\hat{\mathbf{k}}\cdot\mathbf{Q})\delta n(\mathbf{k};\mathbf{Q},\Omega)+2iv_F\hat{\mathbf{k}}\cdot\mathbf{Q}\delta(\varepsilon_{\mathbf{k}})\left[\Phi_{\rm ext}(\mathbf{Q},\Omega)+F_0^s\int\frac{d\hat{\mathbf{k}}'}{S_d}\nu(\hat{\mathbf{k}}';\mathbf{Q},\Omega)\right]=-\frac{1}{\tau}\left[\delta n(\mathbf{k};\mathbf{Q},\Omega)-\int\frac{d\hat{\mathbf{k}}'}{S_d}\delta n(\mathbf{k}';\mathbf{Q},\Omega)\right] \quad (2.10)$$

with $|\mathbf{k}|=|\mathbf{k}'|$, and with the neglect of the angular dependence both of the Landau interaction function and of the impurity scattering probability.

The correct expressions for the total density and current at order t^0 can directly be recovered from Eq. (2.10). To this end, it is convenient to obtain first from that equation the diffusion equation for the average distribution function, defined by

$$\delta\bar{n}(|\mathbf{k}|;\mathbf{Q},\Omega)=\int\frac{d\hat{\mathbf{k}}}{S_d}\delta n(\mathbf{k};\mathbf{Q},\Omega)\ . \quad (2.11)$$

One obtains

$$(-i\Omega+D_0\mathbf{Q}^2)\delta\bar{n}(|\mathbf{k}|;\mathbf{Q},\Omega)=-2\delta(\varepsilon_{\mathbf{k}})D_0\mathbf{Q}^2\left[\Phi_{\rm ext}(\mathbf{Q},\Omega)+F_0^s\int\frac{d\hat{\mathbf{k}}'}{S_d}\nu(\hat{\mathbf{k}}';\mathbf{Q},\Omega)\right], \quad (2.12)$$

where D_0 is the Drude diffusion coefficient. This equation can, in turn, be solved for $\delta\bar{n}$, yielding the total number density

$$\delta n(\mathbf{Q},\Omega)=N_0(\varepsilon_F)\int d\varepsilon_{\mathbf{k}}\delta\bar{n}(|\mathbf{k}|;\mathbf{Q},\Omega)=-\frac{2N_0(\varepsilon_F)}{1+F_0^s}\frac{D_0\mathbf{Q}^2}{D_0\mathbf{Q}^2-i\Omega/(1+F_0^s)}\Phi_{\rm ext}(\mathbf{Q},\Omega)\ . \quad (2.13)$$

Notice, in particular, that the static limit of Eq. (2.13) recovers the correct Fermi-liquid correction to the adiabatic compressibility $\partial n/\partial\mu=2N_0(\varepsilon_F)/(1+F_0^s)$.

The total current can instead be obtained by combining Eqs. (2.10) and (2.12). To leading order in $\mathbf{Q}$ and Ω one gets

$$\mathbf{j}(\mathbf{Q},\Omega)=\int\frac{d\mathbf{k}}{(2\pi)^d}\frac{\mathbf{k}}{m}\delta n(\mathbf{k};\mathbf{Q},\Omega)=2N_0(\varepsilon_F)v_F\int\frac{d\hat{\mathbf{k}}}{S_d}\hat{\mathbf{k}}\nu(\hat{\mathbf{k}};\mathbf{Q},\Omega)=-i\mathbf{Q}\Phi_{\rm ext}(\mathbf{Q},\Omega)2N_0(\varepsilon_F)D_0\ , \quad (2.14)$$

whereby the electric conductivity coincides with its value in the absence of the electron-electron interaction (within the present approximation).

The diffusion equation (2.12) could be obtained directly from the diagrammatic structure, without having to pass through the Boltzmann equation (2.10). Since the latter procedure can be extended to the case of strong disorder, we shall consider it in detail in the next section. For a discussion of the problems encountered in deriving the corresponding Boltzmann equation, we refer instead the reader to the Appendix.

III. KINETIC EQUATION FOR QUASIPARTICLES IN THE PRESENCE OF STRONG DISORDER: A FERMI-LIQUID DESCRIPTION IN TERMS OF THE ENERGY DISTRIBUTION FUNCTION

We begin by reconsidering the derivation of the diffusion equation (2.12) for the energy distribution function of quasiparticles in the weak disorder case (order t^0), by obtaining it directly from the diagrammatic structure. To this end, it is convenient to perform at the outset the resummation of the sequence of diagrams depicted in Fig. 1, with the insertion of the direct ladder

$$L(\mathbf{Q},\Omega)=\frac{1}{2\pi N_0(\varepsilon_F)\tau^2}\frac{1}{D_0\mathbf{Q}^2-i\Omega} \quad (3.1)$$

within each pair of $+-$ single-particle lines. In this way, one is led to introduce a *dynamical amplitude*,[1,2] defined by the equation

$$\Gamma_{\rm eff}(\mathbf{Q},\Omega)=\Gamma_s^\infty-i\Omega 2\pi N_0(\varepsilon_F)^2\tau^2\Gamma_s^\infty\times L(\mathbf{Q},\Omega)\Gamma_{\rm eff}(\mathbf{Q},\Omega)\ , \quad (3.2)$$

where the Ω factor on the right-hand side originates from the restricted range of the frequency integrals. Notice that $\Gamma_{\rm eff}(\mathbf{Q},\Omega)$ reduces to Γ_s^∞ in the static limit $Q/\Omega\to\infty$. Equation (3.2) can be readily solved, to give

$$iN_0(\varepsilon_F)\Gamma_{\rm eff}(\mathbf{Q},\Omega)=iN_0(\varepsilon_F)\Gamma_s^\infty\frac{D_0\mathbf{Q}^2-i\Omega}{D_0\mathbf{Q}^2-i\Omega[1-iN_0(\varepsilon_F)\Gamma_s^\infty]}\ . \quad (3.3)$$

The angular average $\delta\bar{g}$ of the dynamical part of the distribution function can now be obtained as follows:

$$\delta\bar{g}(\omega;\mathbf{Q},\Omega)=-i\Omega 2\delta(\omega)2\pi N_0(\varepsilon_F)\tau^2 L(\mathbf{Q},\Omega)[1-i\Omega 2\pi N_0(\varepsilon_F)^2\tau^2 L(\mathbf{Q},\Omega)\Gamma_{\rm eff}(\mathbf{Q},\Omega)]\Lambda_s^\infty\Phi_{\rm ext}(\mathbf{Q},\Omega)$$
$$=-2\delta(\omega)\frac{i\Omega}{D_0\mathbf{Q}^2-i\Omega[1-iN_0(\varepsilon_F)\Gamma_s^\infty]}\Lambda_s^\infty\Phi_{\rm ext}(\mathbf{Q},\Omega) \quad (3.4)$$

with the neglect of terms which are irrelevant in the hydrodynamic regime. The same result obviously follows upon averaging Eq. (2.5) over the direction $\hat{\mathbf{k}}$.

In the presence of strong disorder (order t^1), the quantities Γ_s^∞ and Λ_s^∞ acquire corrections which are logarithmically singular in two dimensions.[1,2] By labeling with a tilde the relevant quantities corrected to first order in the disorder parameter t, we have in particular that $\tilde{\Lambda}_s^\infty=\Lambda_s^\infty/\zeta,^2$ where $\zeta=N(\varepsilon_F)/N_0(\varepsilon_F)$ specifies the corrections acquired by the single-particle density of states.[4] The direct ladder $L(\mathbf{Q},\Omega)$ is also dressed by the combined action of disorder and interaction in the following way:[1,2]

$$\tilde{L}(\mathbf{Q},\Omega)=\frac{1}{2\pi N_0(\varepsilon_F)\tau^2}\frac{\zeta^2}{D'\mathbf{Q}^2-iz\Omega}, \quad (3.5)$$

where z is a frequency renormalization parameter that turns out to yield the correction to the specific heat over and above its Landau value,[9] while D'/z will soon be identified with the quasiparticle diffusion coefficient. Both z and D' have logarithmically divergent contributions of first order in t in two dimensions.[1,2] The set of diagrams yielding the renormalized direct ladder (3.5) is further considered in the Appendix.

A dynamical amplitude can also be defined in the presence of strong disorder, by replacing Γ_s^∞ and $L(\mathbf{Q},\Omega)$ on the right-hand side of Eq. (3.2) with their corrected counterparts $\tilde{\Gamma}_s^\infty$ and $\tilde{L}(\mathbf{Q},\Omega)$. One obtains

$$iN_0(\varepsilon_F)\tilde{\Gamma}_{\rm eff}(\mathbf{Q},\Omega)$$
$$=iN_0(\varepsilon_F)\tilde{\Gamma}_s^\infty\frac{D'\mathbf{Q}^2-iz\Omega}{D'\mathbf{Q}^2-i\Omega[z-iN_0(\varepsilon_F)\zeta^2\tilde{\Gamma}_s^\infty]} \quad (3.6)$$

in place of Eq. (3.3). By the same token, in the presence of strong disorder Eq. (3.4) becomes

$$\delta\bar{g}(\omega;\mathbf{Q},\Omega)=-2\delta(\omega)\frac{i\Omega}{D'\mathbf{Q}^2-i\Omega[z-iN_0(\varepsilon_F)\zeta^2\tilde{\Gamma}_s^\infty]}\times\zeta^2\tilde{\Lambda}_s^\infty\Phi_{\rm ext}(\mathbf{Q},\Omega), \quad (3.7)$$

which can be conveniently manipulated in the form of a diffusion equation

$$\left[-i\Omega+\frac{D'}{z}\mathbf{Q}^2\right]\delta\bar{g}(\omega;\mathbf{Q},\Omega)$$
$$=-2\delta(\omega)i\Omega\zeta\left[\frac{\zeta}{z}\tilde{\Lambda}_s^\infty\Phi_{\rm ext}(\mathbf{Q},\Omega)+iN_0(\varepsilon_F)\frac{\zeta}{z}\tilde{\Gamma}_s^\infty\bar{\gamma}(\mathbf{Q},\Omega)\right], \quad (3.8)$$

where we have introduced the notation

$$\delta\bar{g}(\omega;\mathbf{Q},\Omega)=2\delta(\omega)\bar{\gamma}(\mathbf{Q},\Omega) \quad (3.9)$$

in analogy with Eq. (2.6).[14]

Notice that the expression within large parentheses on the right-hand side of Eq. (3.8) plays the role of an "effective field" that drives the distribution function $(z/\zeta)\delta\bar{g}(\omega z;\mathbf{Q},\Omega)$, where the ratio ζ/z can be associated with the single-particle renormalization constant a of the Landau theory of Fermi liquids.[10] It can, in fact, be shown[6] that, to linear order in the external field, the quasiparticle distribution function of the ordinary Landau theory of Fermi liquids can be obtained from the diagrammatic structure by disregarding the incoherent part of the final pair of single-particle Green's functions, provided that one consistently drops a factor a in the linear-response expression. Rescaling the frequency argument of $\delta\bar{g}$ from ω to ωz is furthermore a characteristic feature of the interacting theory in the presence of strong disorder, as results from the renormalization-group procedure to integrate out the fast modes.[2]

These remarks suggest that, to linear order in the external field, the quasiparticle energy distribution function $\delta\bar{n}(\varepsilon;\mathbf{Q},\Omega)$ (which includes the static contribution besides the dynamical one) can be identified as follows:

$$\delta\bar{n}(\varepsilon;\mathbf{Q},\Omega)=\frac{z}{\zeta}\delta\bar{g}(\varepsilon z;\mathbf{Q},\Omega)-2\delta(\varepsilon)\left[\frac{\zeta}{z}\tilde{\Lambda}_s^\infty\Phi_{\rm ext}(\mathbf{Q},\Omega)+iN_0(\varepsilon_F)\frac{\zeta}{z}\tilde{\Gamma}_s^\infty\bar{\gamma}(\mathbf{Q},\Omega)\right], \quad (3.10)$$

where ε has the meaning of the energy of a quasiparticle in isolation. Notice that Eq. (3.10) correctly reduces to the angular average of Eq. (2.7) in the weak disorder limit. Notice also that the sum over the spin components has already been performed in Eq. (3.10), the distribution function being actually spin independent for the coupling to a scalar potential.

The definition (3.10) enables us, in turn, to express the "effective field" in terms of $\delta\bar{n}$ itself. To this end, we set

$$\delta\bar{n}(\varepsilon;\mathbf{Q},\Omega)=2\delta(\varepsilon)\bar{\nu}(\mathbf{Q},\Omega) \quad (3.11)$$

in analogy with Eqs. (2.8), and introduce the notation

$$\tilde{F}_0^s=\frac{iN_0(\varepsilon_F)\frac{\zeta^2}{z}\tilde{\Gamma}_s^\infty}{1-iN_0(\varepsilon_F)\frac{\zeta^2}{z}\tilde{\Gamma}_s^\infty} \quad (3.12)$$

in analogy with Eqs. (2.2) and (2.4). We then obtain

$$\frac{\zeta}{z}\tilde{\Lambda}_s^\infty \Phi_{\rm ext}(\mathbf{Q},\Omega)+iN_0(\varepsilon_F)\frac{\zeta}{z}\tilde{\Gamma}_s^\infty \bar{\gamma}(\mathbf{Q},\Omega) = \Phi_{\rm ext}(\mathbf{Q},\Omega)+\tilde{F}_0^s \bar{v}(\mathbf{Q},\Omega) \; , \qquad (3.13)$$

where use has been made of the relationship[4]

$$\frac{\zeta}{z}\tilde{\Lambda}_s^\infty = 1-iN_0(\varepsilon_F)\frac{\zeta^2}{z}\tilde{\Gamma}_s^\infty \; . \qquad (3.14)$$

The identity (3.13) generalizes Eq. (2.9) to the strong disorder case, and shows at the same time that $\tilde{F}_0^s$ given by Eq. (3.12) can be regarded as the relevant interaction function in this case.

The kinetic equation for $\delta\bar{n}(\varepsilon;\mathbf{Q},\Omega)$ can be readily obtained by entering Eqs. (3.10) and (3.13) into Eq. (3.8). It reads

$$\left[-i\Omega+\frac{D'}{z}\mathbf{Q}^2\right]\delta\bar{n}(\varepsilon;\mathbf{Q},\Omega) = -2\delta(\varepsilon)\frac{D'}{z}\mathbf{Q}^2[\Phi_{\rm ext}(\mathbf{Q},\Omega)+\tilde{F}_0^s\bar{v}(\mathbf{Q},\Omega)] \; . \qquad (3.15)$$

Comparison of this result with its counterpart (2.12) for weak disorder identifies

$$\tilde{D}_{\rm qp}=\frac{D'}{z} \qquad (3.16)$$

as the *quasiparticle diffusion coefficient,* which includes (infrared) singular corrections of first order in the disorder parameter t, to be remedied by the renormalization-group methods.[1,2]

Equation (3.15) constitutes the kinetic equation for the energy distribution function of quasiparticles that holds in the presence of strong disorder. This equation has been introduced in Ref. 5 on the basis of the requirement that all known results for the number, spin, and heat density correlation functions could be obtained from it or its variations.[15] We have provided here a first-principles derivation of this equation, by resting on the (renormalized) diagrammatic structure for the two-particle Green's function as well as on the identification (3.10) for the energy distribution function of quasiparticles.

We now recall explicitly how the kinetic equation (3.15) reproduces all known results of the Fermi-liquid picture of strongly disordered electronic systems.

IV. RECOVERING THE MACROSCOPIC TRANSPORT AND THERMAL PROPERTIES OF DISORDERED INTERACTING SYSTEMS FROM THE KINETIC EQUATION

In this section we establish contact with previous work in linear-response theory for disordered systems by showing how the transport equation derived in this paper leads to the accepted corrections to the electric, spin, and heat current at order t^1. To this end, all relevant diffusion coefficients will be compactly expressed in terms of the quasiparticle diffusion coefficient (3.16) and of the renormalized Fermi-liquid amplitudes; these, in turn, will be expressed in terms of the diffusion coefficient D' of Eq. (3.5) and of the parameters z, z_1, and z_2 which are familiar in localization theory.[1–5] We shall not, however, reproduce here the explicit expressions of these parameters in terms of physical quantities such as the temperature. Nor shall we repropose the set of diagrams for the two-particle Green's function which lead to the disorder corrections at order t^1. For these purposes, we shall refer the reader to the relevant literature.

The quasiparticle diffusion equation (3.15) has been derived under the assumption that the density fluctuations of the system couple linearly to an external scalar potential. More general types of coupling can be realized, such as the coupling to an external magnetic field or to a temperature fluctuation. In these cases the coupling depends on the spin σ and on the energy ε of the quasiparticles, respectively, which we have seen to be the relevant quantum numbers in the presence of strong disorder. Equation (3.15) can thus be generalized to include also the magnetic and thermal cases, by retracing the steps we have described in Sec. III for the potential case. The most general kinetic equation for diffusive quasiparticles acquires then the form

$$(-i\Omega+\tilde{D}_{\rm qp}\mathbf{Q}^2)\delta\bar{n}_\sigma(\varepsilon;\mathbf{Q},\Omega)=\frac{\partial n_0(\varepsilon)}{\partial\varepsilon}\tilde{D}_{\rm qp}\mathbf{Q}^2\left[\mathcal{E}_c+\int d\varepsilon'\sum_{\sigma'}\tilde{F}_0(\sigma,\sigma')\delta\bar{n}_{\sigma'}(\varepsilon';\mathbf{Q},\Omega)\right] \; , \qquad (4.1)$$

where $n_0(\varepsilon)$ stands for the equilibrium distribution function of quasiparticles that we take to be given by the Fermi distribution function at low but finite temperature, and where the symbol $\mathcal{E}_c$ within the large parentheses on the right-hand side of Eq. (4.1) specifies the alternative forms of the "external coupling" in the various cases. The structure of Eq. (4.1) is indeed required for a description of quantum transport in disordered systems in terms of quasiparticles.

We now examine the cases of interest separately.

A. Potential coupling

In this case the external coupling $\mathcal{E}_c=\Phi_{\rm ext}(\mathbf{Q},\Omega)$ and, accordingly, the distribution function are spin independent. We can then write

$$\sum_{\sigma'}\tilde{F}_0(\sigma,\sigma')\delta\bar{n}_{\sigma'}(\varepsilon')=\tilde{F}_0^s\delta\bar{n}_\sigma(\varepsilon') \qquad (4.2)$$

on the right-hand side of Eq. (4.1), where

$$\tilde{F}_0^s=\tilde{F}_0(\uparrow,\uparrow)+\tilde{F}_0(\uparrow,\downarrow) \qquad (4.3)$$

coincides with the singlet component of the (isotropic) Landau interaction function entering Eq. (3.15). Recalling that $\partial n_0(\varepsilon)/\partial\varepsilon$ reduces to $-\delta(\varepsilon)$ in the zero-temperature limit, one verifies that Eq. (4.1) reduces indeed to Eq. (3.15) in that limit upon performing the spin summation.

The total number density induced by $\Phi_{\rm ext}(\mathbf{Q},\Omega)$ can be readily obtained by solving the kinetic equation for $\delta\bar{n}_\sigma(\varepsilon)$. One gets

$$\delta n(\mathbf{Q},\Omega)=\tilde{N}_{\rm qp}\int d\varepsilon\sum_\sigma \delta\bar{n}_\sigma(\varepsilon;\mathbf{Q},\Omega)$$
$$=-\frac{2\tilde{N}_{\rm qp}}{1+\tilde{F}^s_0}\frac{\tilde{D}_{\rm qp}(1+\tilde{F}^s_0)\mathbf{Q}^2}{\tilde{D}_{\rm qp}(1+\tilde{F}^s_0)\mathbf{Q}^2-i\Omega}\Phi_{\rm ext}(\mathbf{Q},\Omega) , \quad (4.4)$$

which generalizes Eq. (2.13) to the case of strong disorder. In this expression $\tilde{N}_{\rm qp}$ stands for the quasiparticle density of states (per spin component) at the corresponding Fermi level, which we take to be given by

$$\tilde{N}_{\rm qp}=zN_0(\varepsilon_F) , \quad (4.5)$$

where z is the frequency renormalization parameter introduced in Eq. (3.5) and $N_0(\varepsilon_F)$ is the density of states of the ordinary Landau theory (cf. Sec. II). The choice (4.5), which is required to recover from Eq. (4.4) the correct expression for the density-density correlation function, turns out to be consistent with one's expectation that the renormalization of the electronic specific heat and of the quasiparticle density of states coincide even in the presence of strong disorder.[16]

From Eq. (4.4) we obtain

$$\frac{\partial n}{\partial\mu}=\frac{2\tilde{N}_{\rm qp}}{1+\tilde{F}^s_0},\quad D_n=\tilde{D}_{\rm qp}(1+\tilde{F}^s_0) , \quad (4.6)$$

for the adiabatic compressibility and the density diffusion coefficient, respectively. Rewriting further $\tilde{F}^s_0$ given by Eq. (3.12) in the form

$$\tilde{F}^s_0=\frac{z-z_1}{z_1} \quad (4.7)$$

(with the notation of Ref. 4) and recalling Eq. (4.5), Eq. (4.6) reduces to

$$\frac{\partial n}{\partial\mu}=2N_0(\varepsilon_F)z_1,\quad D_n=\frac{D'}{z_1} , \quad (4.8)$$

which are familiar results from the study of the density-density correlation function.[1,2] The Einstein relation can then be used to express the electrical conductivity in the form

$$\sigma_e=2e^2N_0(\varepsilon_F)D' \quad (4.9)$$

(e being the electronic charge), which differs from the result (2.14) obtained at order t^0 by the replacement of D_0 with the diffusion coefficient D' entering the ladder (3.5).

B. Magnetic coupling

In this case the external coupling $\mathcal{E}_c=\mu_B\sigma H_{\rm ext}(\mathbf{Q},\Omega)$ (μ_B being the Bohr magneton) and, accordingly, the distribution function changes sign by reversing the spin projection σ along the direction of the magnetic field $H_{\rm ext}$. We thus set

$$\delta\bar{n}_\sigma(\varepsilon;\mathbf{Q},\Omega)=\sigma\delta\bar{n}(\varepsilon;\mathbf{Q},\Omega) , \quad (4.10)$$

and write

$$\sum_{\sigma'}\tilde{F}_0(\sigma,\sigma')\delta\bar{n}_{\sigma'}(\varepsilon')=\tilde{F}^a_0\delta\bar{n}_\sigma(\varepsilon') \quad (4.11)$$

on the right-hand of Eq. (4.1), where now

$$\tilde{F}^a_0=\tilde{F}_0(\uparrow,\uparrow)-\tilde{F}_0(\uparrow,\downarrow) \quad (4.12)$$

is the triplet component of the (isotropic) Landau interaction function. In analogy with Eq. (3.12), we can in fact identify from the diagrammatic structure

$$\tilde{F}^a_0=\frac{iN_0(\varepsilon_F)\frac{\zeta^2}{z}\tilde{\Gamma}^\infty_t}{1-iN_0(\varepsilon_F)\frac{\zeta^2}{z}\tilde{\Gamma}^\infty_t} , \quad (4.13)$$

where $\tilde{\Gamma}^\infty_t$ is the triplet counterpart of the scattering amplitude introduced in Sec. III. Notice that Eqs. (4.3) and (4.12) could be used in reverse to extract the Landau parameters $\tilde{F}_0(\uparrow,\uparrow)$ and $\tilde{F}_0(\uparrow,\downarrow)$ from the diagrammatic structure.

The magnetization density can be obtained by entering Eqs. (4.10) and (4.11) into the quasiparticle kinetic equation (4.1) and solving for $\delta\bar{n}$. One gets

$$M(\mathbf{Q},\Omega)=-\mu_B\tilde{N}_{\rm qp}\int d\varepsilon\sum_\sigma\sigma\delta\bar{n}_\sigma(\varepsilon;\mathbf{Q},\Omega)$$
$$=\mu_B^2\frac{2\tilde{N}_{\rm qp}}{1+\tilde{F}^a_0}\frac{\tilde{D}_{\rm qp}(1+\tilde{F}^a_0)\mathbf{Q}^2}{\tilde{D}_{\rm qp}(1+\tilde{F}^a_0)\mathbf{Q}^2-i\Omega}H_{\rm ext}(\mathbf{Q},\Omega) , \quad (4.14)$$

which enables us to identify the spin-spin correlation function. From Eq. (4.14) we obtain for the static susceptibility and the spin diffusion coefficient

$$\chi^{\rm (st)}_{\rm SS}=\mu_B^2\frac{2\tilde{N}_{\rm qp}}{1+\tilde{F}^a_0},\quad D_S=\tilde{D}_{\rm qp}(1+\tilde{F}^a_0) , \quad (4.15)$$

respectively. Moreover, by rewriting $\tilde{F}^a_0$ given by Eq. (4.13) with the notation of Ref. 4, namely,

$$\tilde{F}^a_0=\frac{z-z_2}{z_2} , \quad (4.16)$$

we recover from Eq. (4.15) the familiar results from the study of the spin-spin correlation function:[3,4]

$$\chi^{\rm (st)}_{\rm SS}=2N_0(\varepsilon_F)\mu_B^2z_2 ,\quad D_S=\frac{D'}{z_2} , \quad (4.17)$$

$2N_0(\varepsilon_F)\mu_B^2$ being the Pauli susceptibility.

C. Thermal coupling

The coupling of a quasiparticle to a thermal perturbation can be written in the form $\mathcal{E}_c=-\varepsilon\delta T(\mathbf{Q},\Omega)/T$, where ε stands, as before, for the energy of a quasiparticle in isolation and $\delta T(\mathbf{Q},\Omega)$ represents a small temperature fluctuation about the equilibrium temperature T. In this case, the quasiparticle "effective potential" on the right-hand side of Eq. (4.1) can be neglected by the usual argument, that it is of order T^2 at low temperature.[6] This argument is actually consistent with the (Ward) identity for the heat vertex $\tilde{\Lambda}_Q$,[5]

$$\frac{\zeta}{z}\tilde{\Lambda}_Q=1 , \tag{4.18}$$

that replaces Eq. (3.14) in the thermal case.

The (grand canonical) energy density can be obtained from the knowledge of the (spin-independent) $\delta\tilde{n}_\sigma(\varepsilon)$ that solves the kinetic equation (4.1) for a thermal coupling, in the form

$$\delta K(\mathbf{Q},\Omega)=\tilde{N}_{\rm qp}\int d\varepsilon\, \varepsilon \sum_\sigma \delta\tilde{n}_\sigma(\varepsilon;\mathbf{Q},\Omega) , \tag{4.19}$$

which justifies calling ε the energy of a quasiparticle in isolation. One gets[17]

$$\begin{aligned}\delta K(\mathbf{Q},\Omega)&=2\tilde{N}_{\rm qp}\frac{\tilde{D}_{\rm qp}\mathbf{Q}^2}{\tilde{D}_{\rm qp}\mathbf{Q}^2-i\Omega}\\ &\quad\times\int d\varepsilon(-1)\frac{\partial n_0(\varepsilon)}{\partial\varepsilon}\varepsilon^2\frac{\delta T(\mathbf{Q},\Omega)}{T}\\ &=c_V T\frac{\tilde{D}_{\rm qp}\mathbf{Q}^2}{\tilde{D}_{\rm qp}\mathbf{Q}^2-i\Omega}\delta T(\mathbf{Q},\Omega) ,\end{aligned} \tag{4.20}$$

where

$$c_V=2zN_0(\varepsilon_F)\frac{\pi^2}{3}k_B^2T=zc_V^{(0)} \tag{4.21}$$

is the (renormalized) specific heat[9] over and above its value $c_V^{(0)}$ calculated at order t^0 in the presence of weak disorder. Comparison of the result (4.20) with the general hydrodynamic form for the heat-heat correlation function eventually identifies the quasiparticle diffusion coefficient with the heat diffusion coefficient:[5]

$$D_Q=\tilde{D}_{\rm qp}=\frac{D'}{z} . \tag{4.22}$$

Notice finally that Eqs. (4.21) and (4.22) yield the Wiedemann-Franz law in the presence of interaction and strong disorder, in the form[5]

$$\kappa=c_VD_Q=\frac{\sigma_e}{e^2}\frac{\pi^2}{3}k_B^2T , \tag{4.23}$$

where κ is the thermal conductivity and use has been made of Eq. (4.9). Equation (4.23) thus expresses the fact that the ratio $\kappa/\sigma_e T$ is a universal constant down to the metal-insulator transition.

V. CONCLUDING REMARKS

We have seen that three quantities (namely, the quasiparticle density of states $\tilde{N}_{\rm qp}$ and the symmetric $\tilde{F}_0^s$ and antisymmetric $\tilde{F}_0^a$ Landau interaction parameters—or, alternatively, the three parameters z, z_1, and z_2) besides the diffusion coefficient D' specify the modifications introduced by the interaction both in the transport theory and in the equilibrium properties (as characterized by the static values of the correlation functions). The fact that these results could be obtained via a quasiparticle kinetic equation nicely matches the spirit of the original Landau approach to normal Fermi liquids,[6,12] and thus gives definitive support to the validity of a renormalized Fermi liquid theory for strongly disordered electronic systems that has previously emerged from the study of the correlation functions.[9,10] (Besides, by our method we have been able to reproduce the linear-response results in the weak-coupling limit, thereby relating our work to the linear-response literature.)

We have not mentioned in this paper the critical behavior of the density, spin, and heat diffusion coefficients which results from the dependence of these quantities on an infrared cutoff introduced to regularize the theory. This dependence can ultimately be found by explicit calculation of the relevant terms of order t^1 in the diagrammatic structure, as described in detail in Refs. 1–5.

We comment, finally, on the lack of an "effective" dispersion relation $\varepsilon(\mathbf{k})$ for quasiparticles in the presence of strong disorder that we have assumed in this paper. The intimate goal of a search for a Landau-Boltzmann equation in the presence of strong disorder is to try to provide a microscopic interpretation for the localization mechanism in the presence of interaction, on the same footing of what was achieved in paper I for the noninteracting case. A possible suggestion why this kind of search is bound to fail in the interacting case is given in the Appendix, namely, that a purely phase-space kinetic equation cannot account for the physics of strongly disordered interacting electronic systems since the introduction of collective diffusive modes, which is required beforehand to yield the criticality of the metal-insulator transition, affects the dynamics of the interparticle interaction in a nontrivial way and leads to additional diffusive terms in the kinetic equation that cannot be reabsorbed in a redefinition of its parameters.

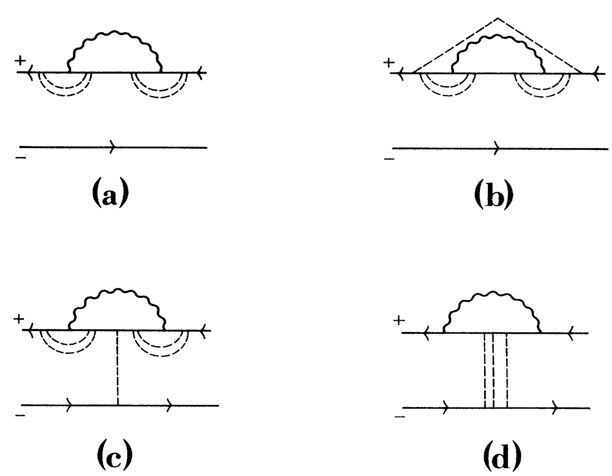

FIG. 2. Diagrams yielding the renormalization of the direct ladder. Full line: impurity averaged noninteracting single-particle Green's function $G_\pm^{(0)}$; broken line: impurity potential scattering; wiggly line: (screened) interparticle interaction v. A sequence of impurity potential lines signifies the insertion of a bare direct ladder [cf. Eq. (3.1) of the text]. The corresponding hole diagrams and the associated Hartree diagrams are not shown for clarity in the figure but are included in the calculation.

APPENDIX: SEARCH FOR A LANDAU-BOLTZMANN EQUATION IN THE PRESENCE OF STRONG DISORDER

In Sec. II we have shown by suitably handling the diagrammatic structure that the dynamical part δg of the distribution function satisfies the integral equation (2.5) in the presence of weak disorder (order t^0). This result, together with the identification (2.7) for the quasiparticle distribution function δn, has led us to the Landau-Boltzmann equation (2.10) with isotropic scattering. It is then natural to question whether the additional terms entering the diagrammatic structure at order t^1 would still allow us to write a kinetic equation similar to (2.5) and whether a meaningful kinetic equation of the Landau-Boltzmann type for the quasiparticle phase-space distribution function would result from it.

To this end, it is necessary to consider explicitly the set of diagrams yielding the renormalized direct ladder (3.5).[1,2] These diagrams are reported in Fig. 2. We will show that the self-energy insertions (a) and (b) modify the terms proportional to δg in Eq. (2.5); that the scattering process (c) modifies instead the collision term proportional to the angular average $\delta\bar{g}$ on the right-hand side of Eq. (2.5); and that the diffusion process (d) introduces additional terms in Eq. (2.5) with $D_0 Q^2$ and $-i\Omega$ acting on δg. Specifically, at order t^1, Eq. (2.5) will be replaced by the following:

$$(-i\Omega c_1 + i v_F \hat{\mathbf{k}}\cdot\mathbf{Q} c_2)\delta g(\hat{\mathbf{k}},\omega;\mathbf{Q},\Omega) + 2i\Omega\delta(\omega)[\tilde{\Lambda}_s^{\infty}\Phi_{\rm ext}(\mathbf{Q},\Omega) + iN_0(\varepsilon_F)\tilde{\Gamma}_s^{\infty}\bar{\gamma}(\mathbf{Q},\Omega)]$$
$$= -\frac{1}{\tilde{\tau}}\left[\delta g(\hat{\mathbf{k}},\omega;\mathbf{Q},\Omega) - \int\frac{d\hat{\mathbf{k}}'}{S_d}\delta g(\hat{\mathbf{k}}',\omega;\mathbf{Q},\Omega)\right] + (i\Omega c_3 - D_0 Q^2 c_4)\int\frac{d\hat{\mathbf{k}}'}{S_d}\delta g(\hat{\mathbf{k}}',\omega;\mathbf{Q},\Omega)\ , \quad \text{(A1)}$$

where use has been made of the notation (3.9). In this equation, the coefficients

$$c_1 = c_2 = 1 + I_1 = \frac{1}{\zeta},\quad c_3 = I_1 - I_3 = \frac{z}{\zeta} - 1,\quad c_4 = I_1 - I_2 = \frac{D'}{\zeta D_0} - 1\ , \quad \text{(A2)}$$

and the effective inverse collision time

$$\frac{1}{\tilde{\tau}} = \frac{1}{\tau}(1 + I_1) \quad \text{(A3)}$$

are expressed in terms of the logarithmically divergent integrals I_1, I_2, and I_3 defined below, which are familiar in localization theory.[1,2] In addition the above coefficients satisfy the constraints

$$c_1 + c_3 = \frac{z}{\zeta^2},\quad c_4 + \frac{\tilde{\tau}}{\tau}c_2^2 = \frac{1}{\zeta^2}\frac{D'}{D_0}\ , \quad \text{(A4)}$$

thereby ensuring that Eq. (3.8) for the angular average $\delta\bar{g}$ is correctly recovered from Eq. (A1).

To prove Eq. (A1), we consider first the self-energy insertions (a) and (b) of Fig. 2 together with the corresponding hole diagrams. To lowest significant order in the disorder parameter t, they give the following contribution to the last pair of $+-$ single-particle lines that terminates the diagrams of Fig. 1 on the left side:[18]

$$\int\frac{d\varepsilon_{\mathbf{k}}}{2\pi}G_+\left[\mathbf{k}+\frac{\mathbf{Q}}{2},\omega+\frac{\Omega}{2}\right]G_-\left[\mathbf{k}-\frac{\mathbf{Q}}{2},\omega-\frac{\Omega}{2}\right]$$
$$\cong\int\frac{d\varepsilon_{\mathbf{k}}}{2\pi}G_+^{(0)}\left[\mathbf{k}+\frac{\mathbf{Q}}{2},\omega+\frac{\Omega}{2}\right]G_-^{(0)}\left[\mathbf{k}-\frac{\mathbf{Q}}{2},\omega-\frac{\Omega}{2}\right]$$
$$+\frac{1}{\tau^2}\int^{\Lambda}\frac{d\mathbf{q}}{(2\pi)^d}\int_{\Omega/2}^{\Lambda}\frac{d\omega'}{2\pi}\frac{i v(\mathbf{q},\omega')}{(D_0 q^2 - i\omega')^2}\int\frac{d\varepsilon_{\mathbf{k}}}{2\pi}G_+^{(0)}\left[\mathbf{k}+\frac{\mathbf{Q}}{2},\omega+\frac{\Omega}{2}\right]$$
$$\times\left[G_+^{(0)}\left[\mathbf{k}+\frac{\mathbf{Q}}{2},\omega+\frac{\Omega}{2}\right]G_-^{(0)}\left[\mathbf{k}+\frac{\mathbf{Q}}{2}-\mathbf{q},\omega+\frac{\Omega}{2}-\omega'\right]\right.$$
$$+G_-^{(0)}\left[\mathbf{k}-\frac{\mathbf{Q}}{2},\omega-\frac{\Omega}{2}\right]G_+^{(0)}\left[\mathbf{k}-\frac{\mathbf{Q}}{2}-\mathbf{q},\omega-\frac{\Omega}{2}+\omega'\right]$$
$$\left.-i\tau G_+^{(0)}\left[\mathbf{k}+\frac{\mathbf{Q}}{2},\omega+\frac{\Omega}{2}\right]+i\tau G_-^{(0)}\left[\mathbf{k}-\frac{\mathbf{Q}}{2},\omega-\frac{\Omega}{2}\right]\right]G_-^{(0)}\left[\mathbf{k}-\frac{\mathbf{Q}}{2},\omega-\frac{\Omega}{2}\right]$$
$$\cong\tau(1-I_1)[1+i\Omega\tau - i\tau v_F\hat{\mathbf{k}}\cdot\mathbf{Q} - \tau^2 v_F^2(\hat{\mathbf{k}}\cdot\mathbf{Q})^2]$$
$$\cong\frac{1-I_1}{\frac{1}{\tau}+i v_F\hat{\mathbf{k}}\cdot\mathbf{Q}-i\Omega}\ . \quad \text{(A5)}$$

Here, $|\omega| < \Omega/2$, Λ is an ultraviolet cutoff needed to regularize the theory, and

$$I_1 = -2\int^{\Lambda}\frac{d\mathbf{q}}{(2\pi)^d}\int_{\Omega/2}^{\Lambda}\frac{d\omega'}{2\pi}\frac{i\nu(\mathbf{q},\omega')}{(D_0\mathbf{q}^2-i\omega')^2} \ . \tag{A6}$$

Notice that Eq. (A5) differs from its noninteracting counterpart by the factor $1-I_1$ which gives the renormalization ζ of the single-particle density of states.[4] Notice also that this factor affects all terms proportional to δg in the kinetic equation (A1).

The contribution to Eq. (A1) of the scattering process (c) of Fig. 2, together with the corresponding hole diagram, results instead from evaluating the following expression:[18]

$$\begin{aligned}
&\int\frac{d\varepsilon_{\mathbf{k}}}{2\pi}G_{+}^{(0)}\left[\mathbf{k}+\frac{\mathbf{Q}}{2},\omega+\frac{\Omega}{2}\right]G_{-}^{(0)}\left[\mathbf{k}-\frac{\mathbf{Q}}{2},\omega-\frac{\Omega}{2}\right]\\
&\quad\times\int\frac{d\mathbf{k}'}{(2\pi)^d}G_{+}^{(0)}\left[\mathbf{k}'+\frac{\mathbf{Q}}{2},\omega+\frac{\Omega}{2}\right]G_{-}^{(0)}\left[\mathbf{k}'-\frac{\mathbf{Q}}{2},\omega-\frac{\Omega}{2}\right]\\
&\quad\quad\times\frac{1}{2\pi N_0(\varepsilon_F)\tau^3}\int^{\Lambda}\frac{d\mathbf{q}}{(2\pi)^d}\int_{\Omega/2}^{\Lambda}\frac{d\omega'}{2\pi}\frac{i\nu(\mathbf{q},\omega')}{(D_0\mathbf{q}^2-i\omega')^2}\\
&\quad\quad\quad\times\left[G_{-}^{(0)}\left[\mathbf{k}+\frac{\mathbf{Q}}{2}-\mathbf{q},\omega+\frac{\Omega}{2}-\omega'\right]G_{-}^{(0)}\left[\mathbf{k}'+\frac{\mathbf{Q}}{2}-\mathbf{q},\omega+\frac{\Omega}{2}-\omega'\right]\right.\\
&\quad\quad\quad\quad\left.+G_{+}^{(0)}\left[\mathbf{k}-\frac{\mathbf{Q}}{2}-\mathbf{q},\omega-\frac{\Omega}{2}+\omega'\right]G_{+}^{(0)}\left[\mathbf{k}'-\frac{\mathbf{Q}}{2}-\mathbf{q},\omega-\frac{\Omega}{2}+\omega'\right]\right]\\
&\cong\frac{1}{\frac{1}{\tau}+iv_F\hat{\mathbf{k}}\cdot\mathbf{Q}-i\Omega}\frac{1}{\tau}I_1\int\frac{d\hat{\mathbf{k}}'}{S_d}\int\frac{d\varepsilon_{\mathbf{k}}'}{2\pi}G_{+}^{(0)}\left[\mathbf{k}'+\frac{\mathbf{Q}}{2},\omega+\frac{\Omega}{2}\right]G_{-}^{(0)}\left[\mathbf{k}'-\frac{\mathbf{Q}}{2},\omega-\frac{\Omega}{2}\right] ,
\end{aligned}\tag{A7}$$

which differs from the contribution of the single-impurity scattering process by the presence of the factor I_1. A combination of both scattering processes leads then to the term on the right-hand side of Eq. (A1) where the effective inverse collision time (A3) multiplies the angular average $\delta\bar{g}$.

The diagrams considered thus far would keep at order t^1 the same functional form of the kinetic equation for δg which holds at order t^0, apart from a renormalization of its coefficients. The diffusion process (d) of Fig. 2 modifies, however, this functional form by introducing two additional terms where $D_0\mathbf{Q}^2$ and $-i\Omega$ act on $\delta\bar{g}$. These new terms result from evaluating the following expression:[18]

$$\begin{aligned}
&\int\frac{d\varepsilon_{\mathbf{k}}}{2\pi}G_{+}^{(0)}\left[\mathbf{k}+\frac{\mathbf{Q}}{2},\omega+\frac{\Omega}{2}\right]G_{-}^{(0)}\left[\mathbf{k}-\frac{\mathbf{Q}}{2},\omega-\frac{\Omega}{2}\right]\\
&\quad\times\int\frac{d\mathbf{k}'}{(2\pi)^d}G_{+}^{(0)}\left[\mathbf{k}'+\frac{\mathbf{Q}}{2},\omega+\frac{\Omega}{2}\right]G_{-}^{(0)}\left[\mathbf{k}'-\frac{\mathbf{Q}}{2},\omega-\frac{\Omega}{2}\right]\\
&\quad\quad\times\frac{1}{2\pi N_0(\varepsilon_F)\tau^2}\int^{\Lambda}\frac{d\mathbf{q}}{(2\pi)^d}\left[\int_{-\Lambda}^{\omega+\Omega/2}\frac{d\omega'}{2\pi}\frac{i\nu(\mathbf{q},\omega')}{D_0(\mathbf{q}-\mathbf{Q})^2+i(\omega'-\Omega)}\right.\\
&\quad\quad\quad\quad\times G_{+}^{(0)}\left[\mathbf{k}+\frac{\mathbf{Q}}{2}-\mathbf{q},\omega+\frac{\Omega}{2}-\omega'\right]G_{+}^{(0)}\left[\mathbf{k}'+\frac{\mathbf{Q}}{2}-\mathbf{q},\omega+\frac{\Omega}{2}-\omega'\right]\\
&\quad\quad\quad+\int_{\omega-\Omega/2}^{\Lambda}\frac{d\omega'}{2\pi}\frac{i\nu(\mathbf{q},\omega')}{D_0(\mathbf{q}+\mathbf{Q})^2-i(\omega'+\Omega)}\\
&\quad\quad\quad\quad\left.\times G_{-}^{(0)}\left[\mathbf{k}-\frac{\mathbf{Q}}{2}-\mathbf{q},\omega-\frac{\Omega}{2}-\omega'\right]G_{-}^{(0)}\left[\mathbf{k}'-\frac{\mathbf{Q}}{2}-\mathbf{q},\omega-\frac{\Omega}{2}-\omega'\right]\right]\\
&\quad\cong\frac{1}{\frac{1}{\tau}+iv_F\hat{\mathbf{k}}\cdot\mathbf{Q}-i\Omega}[D_0\mathbf{Q}^2(I_2-I_1)-i\Omega(I_3-I_1)]\\
&\quad\quad\times\int\frac{d\hat{\mathbf{k}}'}{S_d}\int\frac{d\varepsilon_{\mathbf{k}'}}{2\pi}G_{+}^{(0)}\left[\mathbf{k}'+\frac{\mathbf{Q}}{2},\omega+\frac{\Omega}{2}\right]G_{-}^{(0)}\left[\mathbf{k}'-\frac{\mathbf{Q}}{2},\omega-\frac{\Omega}{2}\right]
\end{aligned}\tag{A8}$$

to the leading singular order, with

$$I_2 = -\frac{8}{d}\int^{\Lambda}\frac{d\mathbf{q}}{(2\pi)^d}\int_{\Omega/2}^{\Lambda}\frac{d\omega'}{2\pi}D_0\mathbf{q}^2\frac{i\nu(\mathbf{q},\omega')}{(D_0\mathbf{q}^2-i\omega')^3}\ , \tag{A9}$$

$$I_3 = \frac{1}{\pi}\int^{\Lambda}\frac{d\mathbf{q}}{(2\pi)^d}\frac{\nu(\mathbf{q},\omega'=0)}{D_0\mathbf{q}^2-i\Omega}\ . \tag{A10}$$

This proves Eq. (A1).

The occurrence of diffusion terms in Eq. (A1) at order t^1, which did not appear in Eq. (2.5) at order t^0, can be interpreted as a signal that the search for a Landau-Boltzmann equation in the presence of strong disorder is bound to fail. Nonetheless, we may proceed up to the end of the argument and define a quasiparticle phase-space distribution function in analogy to Eq. (3.10) as follows:

$$\delta n(\mathbf{k};\mathbf{Q},\Omega) = \frac{z}{\zeta}\delta g(\hat{\mathbf{k}},z\tilde{\varepsilon}_{\mathbf{k}};\mathbf{Q},\Omega) - 2\delta(\tilde{\varepsilon}_{\mathbf{k}})\left[\frac{\zeta}{z}\tilde{\Lambda}_s^{\infty}\Phi_{\text{ext}}(\mathbf{Q},\Omega) + iN_0(\varepsilon_F)\frac{\zeta}{z}\tilde{\Gamma}_s^{\infty}\overline{\gamma}(\mathbf{Q},\Omega)\right]\ , \tag{A11}$$

where $\tilde{\varepsilon}_{\mathbf{k}}$ has the meaning of the $\mathbf{k}$-dependent energy of a quasiparticle with density of states given by Eq. (4.5). Combining the definition (A11) with Eq. (A1) and taking into account the identity (3.13) leads eventually to the following equation:

$$\begin{aligned}(-i\Omega + iv_F\hat{\mathbf{k}}\cdot\mathbf{Q})\delta n(\mathbf{k};\mathbf{Q},\Omega) &+ 2\delta(\tilde{\varepsilon}_{\mathbf{k}})(iv_F\hat{\mathbf{k}}\cdot\mathbf{Q}+D_0\mathbf{Q}^2c_4)[\Phi_{\text{ext}}(\mathbf{Q},\Omega)+\tilde{F}_0^s\overline{\nu}(\mathbf{Q},\Omega)] \\ &= -\frac{1}{\tau}\left[\delta n(\mathbf{k};\mathbf{Q},\Omega) - \int\frac{d\hat{\mathbf{k}}'}{S_d}\delta n(\mathbf{k}';\mathbf{Q},\Omega)\right] + (i\Omega c_3 - D_0\mathbf{Q}^2c_4)\int\frac{d\hat{\mathbf{k}}'}{S_d}\delta n(\mathbf{k}';\mathbf{Q},\Omega)\end{aligned} \tag{A12}$$

with $|\mathbf{k}|=|\mathbf{k}'|$. Notice that the mixed ballistic and diffusive character of the kinetic equation is even more pronounced in Eq. (A12) than in Eq. (A1) since it now appears also in the driving term. We think, however, that this feature can hardly be avoided due to the necessity of introducing the diffusive modes to handle the physics of strongly disordered electronic systems. In the presence of interaction these diffusive modes give, in fact, an additional dynamics to the effective interparticle interaction which depends explicitly on D_0.

Although the correct diffusion equation (3.15) with diffusion coefficient (3.16) results by taking the spherical average of Eq. (A12), this latter equation fails to give the correct expression for the particle current and it is therefore not consistent with the continuity equation. Upon integrating Eq. (A12) over the wave vector $\mathbf{k}$ we in fact obtain

$$i\mathbf{Q}\cdot\left[\frac{\tilde{D}_{\text{qp}}}{D_0}\int\frac{d\mathbf{k}}{(2\pi)^d}\frac{\mathbf{k}}{m}\delta n(\mathbf{k};\mathbf{Q},\Omega)\right] - i\Omega\int\frac{d\mathbf{k}}{(2\pi)^d}\delta n(\mathbf{k};\mathbf{Q},\Omega) = 0\ , \tag{A13}$$

m being the mass of a *bare* particle. From this result one can draw the conclusion that the failure to fulfill the continuity equation definitely spoils the quasiparticle phase-space distribution function of any meaning, as we have consistently assumed throughout this paper. We have accordingly dismissed any further search for a Landau-Boltzmann equation at order t^1.

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[14]Our Eq. (3.8) bears some analogy with Eq. (13) of Ref. 11, since in Fourier space the driving term is proportional to the

external frequency Ω. It is, however, clear from our derivation that Eq. (3.8) cannot be considered as the kinetic equation for diffusing quasiparticles.

[15]The kinetic equation for diffusing quasiparticles given in the first of Refs. 5 [cf. Eq. (5) therein] can be mapped onto Eq. (3.15) of the present paper by taking its time and space Fourier transform, provived that the term containing the Landau interaction function is interpreted as including a convolution in the time variable. Recall, in fact, that (at zero temperature) the external frequency Ω is taken in the interacting case, too, as the infrared cutoff of the theory that regularizes the singular corrections at first order in the disorder parameter t. As a consequence, an Ω dependence has to be understood in the Landau interaction parameter $\tilde{F}_0^s$ entering Eq. (3.15). [Notice, in addition, a misprint for the sign of the last term on the right-hand side of Eq. (5) in Ref. 5, and that the spin indices have been explicitly kept in that equation.]

[16]Notice that, from Eq. (4.5) and from the identification we made in Sec. III of the ratio ζ/z with the single-particle renormalization constant, expressions like Eqs. (3.12)–(3.14) turn out to have the correct factors in front of the amplitude $\tilde{\Gamma}_s^\infty$ and of the vertex $\tilde{\Lambda}_s^\infty$, as one would expect on the basis of the ordinary Landau theory of Fermi liquids (cf. Ref. 6).

[17]The total number density $\delta n(\mathbf{Q},\Omega)$ induced by a thermal coupling could also be obtained from the solution of the kinetic equation (4.1), provided one allows the quasiparticle density of states and the diffusion coefficient to depend on the quasiparticle energy ε. In this way, one can recover the double-pole structure of the mixed number-heat correlation function in the presence of the electron-electron interaction [M. Fabrizio, C. Castellani, and G. Strinati, Phys. Rev. B **43**, 11 088 (1991); for the corresponding analysis in the noninteracting case, see C. Castellani, C. Di Castro, M. Grilli, and G. Strinati, Phys. Rev. B **37**, 6663 (1988)].

[18]Taking into account the associated Hartree diagrams modifies the results by multiplying the integrals I_1, I_2, and I_3 by a numerical factor [cf. Refs. 1 and 2].