Spectral Representation and Moments of a Green's function

Deriving the spectral representation

$$\left\langle\left\{c_k(t),c_k^\dagger(0)\right\}\right\rangle=\sum_{nm}e^{-\beta\epsilon_n}\left\{\langle n\left|c_k(t)\right|m\rangle\left\langle m\left|c_k^\dagger(0)\right|n\right\rangle+\left\langle n\left|c_k^\dagger(0)\right|m\right\rangle\langle m\left|c_k(t)\right|n\rangle\right\} \quad (1)$$

$$\begin{aligned}\int_{-\infty}^{\infty}dte^{i\omega t}\left\langle\left\{c_k(t),c_k^\dagger(0)\right\}\right\rangle &= \sum_{nm}e^{-\beta\epsilon_n}2\pi\delta(\omega-(\epsilon_m-\epsilon_n))\left|\left\langle m\left|c_k^\dagger\right|n\right\rangle\right|^2\\ &\quad+\sum_{nm}e^{-\beta\epsilon_n}2\pi\delta(\omega-(\epsilon_n-\epsilon_m))\left|\langle m\left|c_k\right|n\rangle\right|^2\\ &= 2\pi A(k,\omega)\end{aligned}$$

Deriving the moments of the spectra using the equation of motion

$$i\frac{\partial}{\partial t}c_k=[c_k,H] \quad (2)$$

$$\begin{aligned}2\pi A(k,\omega)\omega^p &= \int_{-\infty}^{\infty}\frac{1}{i^p}\left[\frac{\partial^p}{\partial t^p}e^{i\omega t}\right]\left\langle\left\{c_k(t),c_k^\dagger(0)\right\}\right\rangle dt\\ &= \int_{-\infty}^{\infty}e^{i\omega t}\left\langle\left\{\left[[c_k,H],\overset{ptimes}{\cdots},H\right],c_k^\dagger(0)\right\}\right\rangle dt\end{aligned}$$

Therefore

$$\int_{-\infty}^{\infty}d\omega A(k,\omega)\omega^p=\left\langle\left\{\left[[c_k,H],\overset{ptimes}{\cdots},H\right],c_k^\dagger(0)\right\}\right\rangle \quad (3)$$

and in particular

$$\int_{-\infty}^{\infty}d\omega A(k,\omega)\omega^{p+q}=\left\langle\left\{\left[[c_k,H],\overset{ptimes}{\cdots},H\right],\left[H,\overset{qtimes}{\cdots}\left[H,c_k^\dagger\right]\right]\right\}\right\rangle \quad (4)$$