Problem Set 4: Application of the bosonization method.

Consider a spinless 1d electron gas of right movers only described by

$$H_o = \sum_k k a_k^+ a_k = \int \psi(x)^\dagger \frac{d}{idx} \psi(x)$$

(right movers only)

Add to it the interaction with a single degree of freedom (describing a deep core electron) $H_1 = E_f f^+ f$

The interaction between the conduction electrons and the f level is given by

sitting at the origin; $H_2 = h_2(f^+ f - 1)$. With $h_2 = \frac{v}{L} \sum_{kk'} a_k^+ a_{k'}$, $H_{2=} v\psi(0)^\dagger \psi(0)(f^+ f - 1)$ ($E_f << 0$) and assume that v is very small compared to the bandwith , L is the size of the system. We shall see however that we are not allowed to do perturbation theory in v.

a) Write $H = H_o + H_1 + H_2$ in terms of $f^+ f$, and bose operators, i.e. bosonize the conduction electrons.

(Write H_0 in terms of ρ_q like we did in class, and then write $\psi(0)$ in terms of bose operators)

b) Notice that $[f^+ f, H] = 0$. Describe the eigenstates of the hamiltonian when $f^+ f = 0$ and $f^+ f = 1$. What is the ground state of the system?

c) Find a canonical transformation U that transforms the bosonized versions of h_2 into H_o , i.e.$U^\dagger H_o U = H_o - h_2$

d) Calculate the Greens function of the heavy hole $G(t) =< o|f^+(t) f(o)|o >$ at long times. ($|o >$ is the ground state of the system) Hint: use the results of c.

e) Take the Fourrier transform of d, what is the physical meaning of your results.

NOTICE I ELIMINATED THE LAST QUESTION (SECTION f) IN THE HOMEWORK.