Problem Set 4, Due After Thanksgivings Break

Problem 1:

$$H_0 = \sum_{k\sigma} \varepsilon_k c^\dagger_{k\sigma} c_{k\sigma}$$

$$\epsilon_k = \frac{k^2}{2m} - \mu$$

$$\mid \psi_0\rangle = \prod_{k<k_F} c^\dagger_{k\uparrow} c^\dagger_{k\downarrow} \mid 0\rangle$$

$$\rho_q \equiv \sum_{k\sigma} \quad c^\dagger_{k+q\sigma} c_{k\sigma}$$

$$\Delta^\dagger q \equiv \sum_k c^\dagger_{-k+q\uparrow} c^\dagger_{k\downarrow}$$

a) zero temperature.

Calculate the Fourier transform of $G_\rho(t,q)$, $G_p(t,q)$ in time.

$$G_\rho(t,q) = (-i)\langle T(\rho_q(t)\rho_{-q}(0))\rangle$$

$$G_p(t,q) = (-i)\langle T(\Delta_q(t){\Delta^\dagger}_q(0))\rangle$$

write an expression it will involve a k - integration consider the case of d dimensions can you evaluate some of the integrals in d=1,2, or 3?

Problem 2 Repeat the first problem at finite temperature Finite temperature - if τ is the imaginary time it is convenient to use Fourier series

$$G_\rho(i\nu_n, g) = \int_o^\beta e^{i\nu_n\tau} G_\rho(\tau, q) d\tau$$

$$G_p(i\nu_n, q) = \int_o^\beta e^{i\nu_n}\tau G_p(\tau, q) d\tau.$$

Evaluate them in 3 and 2 dimensions. 1

Now G_ρ and G_p are defined by

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$$G_\rho(\tau,q) = \frac{1}{Z} tr e^{-\beta(H_o-\mu N)} \rho_q(\tau)\rho_{-q}(o)$$

$$G_\rho(\tau,q) = \frac{1}{Z} tr e^{-\beta(H_o-\mu N)} \Delta_q(\tau)\Delta^\dagger q(o)$$

Problem 3: Evaluate the following Matsubara sums

$$g(\tau = 0^+) = T\sum_n \frac{e^{i\omega_n 0^+}}{i\omega_n - \varepsilon_k}$$

and

$$g(\tau = 0^-) = T\sum_n \frac{e^{i\omega_n 0^-}}{i\omega_n - \varepsilon_k},$$

where ω_n is fermionic Matsubara frequency, i.e. $\omega_n = \frac{(2n+1)\pi}{\beta}$. Interpret the result why the two sums differ