Phase Space Interchange

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Last year Kwang-Je Kim in association with Andrew Sessler proposed a method to accomplish phase space interchange between transverse and longitudinal degrees of freedom[1] and earlier this year a second related publication elaborated further[2]. A few years ago, Max Cornacchia and Paul Emma made a significant step in this direction when they suggested the insertion of a deflecting mode cavity midway through a magnetic chicane[3]. The Cornacchia-Emma approach was already sufficiently interesting for us to begin design of modifications to the photoinjector at Fermilab for a demonstration experiment; Kim's improved method provides even more incentive. Actually, about a half-dozen years ago Alexander Zholents, a member of our advisory committee, pointed out that we should look into this interchange mode as a corollary to the "flat-beam" transformation that we were measuring at that time; in retrospect, we should have acted sooner.

In the spirit of single particle linear dynamics, phase space interchange between the pairs x, x' and z, $z' \equiv \delta p/p$ can be accomplished by an assembly described by a 4-by-4 matrix of the form

$$M = \begin{pmatrix} 0 & B \\ C & 0 \end{pmatrix}, \tag{1}$$

where B and C are 2-by-2 matrices. Kwang-Je's discovery was that if he reversed the bend direction of the second dogleg in the Cornacchia-Emma chicane, he arrived at a matrix as shown in Eq. 1, while the strength of the deflecting mode cavity remained the same.

Naturally, I began to wonder if there were other assemblies that also produced the transfer matrix of Eq. 1. Some details of this search may be found in the Appendix. Here, I would only like to comment on alternatives that I thought of as potential competitors. First, consider a single dogleg, and at the midpoint of the drift separating the two bends, place a quadrupole that focuses one bend on the other and at that point co-locate the deflecting mode cavity. As thin lenses, the cavity and quadrupole matrices commute and so the order does not matter in this approximation.

If the magnitude of the bend angle of each magnet is θ, and the spacing between bends is 2ℓ, then the momentum dispersion at entry to the cavity is $\theta\ell$, and as in the citations above, the energy change versus transverse displacement in the cavity is chosen to cancel the dispersion. As noted in the preceding paragraph, the focal length of the quadrupole is $\ell/2$. The resulting transfer matrix is

$$M = \left(\begin{array}{cccc} 0 & 0 & 1/\theta & 0 \\ 0 & 0 & 1/(\ell\theta) & \theta \\ \theta & 0 & 0 & 0 \\ 1/(\ell\theta) & 1/\theta & 0 & \end{array} \right). \tag{2}$$

Neither the deflecting mode cavity nor a quadrupole are “thin” on the scale of our photoinjector, so the implementation associated with this approach would have quadrupoles on both sides of the cavity.

This arrangement leads to the same desired result as the Kim solution, but at the use of more lateral space in the limited confines of our photoinjector enclosure. Then there are combinations of the two concepts, such as an initial dogleg *a la* Kim to provide zero slope to the momentum dispersion at entry to the cavity, followed by a quadrupole-drift-bend combination to complete the production of the transformation characterized in Eq. 1. Ray Fliller has also looked at the ways of approaching the transformation[4]. This has been a very useful discussion, but at the end we have adopted the Kim geometry.

I should remark that the requirements on the deflecting mode RF structure are the same, fortunately, in all these variants. I say fortunately, because the device has been under construction for some time now. The derivative of energy change versus transverse displacement is given by the ratio of beam energy to momentum dispersion at the cavity location. Our small beam energy of 15 MeV and a dogleg of 1/3 m offset leads to a rather reasonable requirement of 45 keV/mm on the deflecting mode cavity. Higher beam energy would certainly be preferable in order to make space charge effects less of a consideration, but that circumstance must await another facility.

The initial demonstration is intended to be the concluding aspect of Tim Koeth’s doctoral dissertation, and further information may be found at his website[5]. With the adventurous parameters associated with new project proposals, further development is likely and participants are both welcome and essential. The attached appendix outlines the thin lens mathematics of the two solutions.

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References

[1] Kwang-Je Kim and Andrew M. Sessler, ”Transverse-Longitudinal Phase Space Manipulation and Correlations”, Proceedings of International Workshop on Beam Cooling and Related Topics, September 18-23, 2005, Galena, IL.

[2] P. Emma, Z. Huang, K.-J. Kim and P. Piot, Phys. Rev. ST Accel. Beams 9, 100702 (2006)

[3] M. Cornacchia and P. Emma, Phys. Rev. ST Accel. Beams 5, 084001 (2002)

[4] R. P. Fliller, Beams-doc#2271-v1@http://beamdocs.fnal.gov/AD-public/DocDB/Search

[5] koeth homepage at www.physics.rutgers.edu/