Physics 601: Solid State Physics I
Problem Set #2, due Monday, September 20, 2004

This is the first of two problems in PS#2. The second, on crystal structure, will be added on Monday, September 13.

Problem 1: The Free and Independent Electron Gas in Two Dimensions (A&M p. 53)

(a) What is the relation between n and k_f in two dimensions?

(b) What is the relation between k_f and r_s in two dimensions?

(c) Show that in two dimensions the free electron density of states g(e) is a constant independent of e for e>0, and 0 for e<0. What is the constant?

(d) Show that because g(e) is constant, every term in the Sommerfeld expansion for n vanishes expect the T=0 term. Deduce that mu=e_f at any temperature.

(e) Using the relation between n, g and mu

$$n = \int_{-\infty}^{\infty} d\varepsilon\, g(\varepsilon) f(\varepsilon)$$

deduce that when g(e) is as in (c), then

$$\mu + k_B T \ln\left(1 + e^{-\mu/k_B T}\right) = \varepsilon_F$$

(f) Estimate from the expression in (e) the amount by which mu differs from e_f. Comment on the numerical significance of this "failure" of the Sommerfeld expansion, and on the mathematical reasons for the "failure."