BEFORE CLASS: Play with toys – how is the energy stored?
Do “check up exercise” for today
ask all your questions as I have to leave right at the end of class

SET TIME FOR FIRST HOUR EXAM:
WED FEB 26 in class (HW1-4)

3rd HOMEWORK ASSIGNMENT IS DUE Wed
4th HW DUE IN CLASS Wed the 19th

class web site
http://www.physics.rutgers.edu/ugrad/272
Find potential from electric field

Find potential directly from charge distribution
Spherically symmetric charge distribution

\[ \rho(\vec{r}) = f(r) \]

Divide charge distribution into spherical shells
Sum or integrate the potentials from the shells

Need to know the potential from a shell from charge distribution, or from electric field?
Outside the shell – electric field is the same as if the charge on the shell $Q$ were concentrated at the center.

So the potential is the same as the potential of a point charge $Q$.

Inside the shell – electric field is zero. The potential is the same as the potential at the surface of the sphere.
Potential of a spherical shell (charge $Q$, radius $R$)

For $r > R$

\[ \vec{E}(\vec{r}) = kQ / r^2 \hat{r} \]

\[ \phi(\vec{r}) = \int_{r'=r}^{r'=\infty} kQ / r'^2 \, dr' = kQ / r \]

Same as for point charge $Q$

(reference point is at $r = \infty$)

For $r < R$

$E$ is zero $\Rightarrow \phi$ is constant

\[ \phi(\vec{r}) = kQ / R \]
Potential of a uniformly charged wire segment $\lambda$
Potential of a uniformly charged wire segment $\lambda$

Slice up into tiny segments and sum $d\phi = (k/r)\ dQ$

Uniform wire on the line

$$\phi(x) = \int_{s=x_L}^{s=x_R} \frac{k\lambda ds}{|s-x|}$$

For points on the line to the left of the wire

$$V(x) = k\lambda \ln\left(\frac{x-x_R}{x-x_L}\right)$$
Figure 24-40 shows a thin rod with a uniform charge density of 2.00 $\mu$C/m. Evaluate the electric potential at point $P$ if $d = D = L/4.00$. 

```
d
  x
```

```
\text{Rod}
```

```
D
```

```
L
```

$P$
32. A nonuniform linear charge distribution given by \( \lambda = bx \), where \( b \) is a constant, is located along an \( x \) axis from \( x = 0 \) to \( x = 0.20 \) m. If \( b = 20 \) nC/m\(^2\) and \( V = 0 \) at infinity, what is the electric potential at (a) the origin and (b) the point \( y = 0.15 \) m on the \( y \) axis?
Potential of a uniformly charged wire $\lambda$

Slice up into tiny segments and sum $d\phi = (k/r) \ dQ$

Uniform wire in circular arc $
\phi$ at center of the circle $P$

$r=R$ for ALL SEGMENTS

$$\phi = \int \frac{k dQ}{R} = \frac{k}{R} \int dQ = \frac{kQ}{R}$$
Potential of a uniformly charged wire $\lambda$

Slice up into tiny segments and sum $d\phi = (k/r) \, dQ$

**Uniform ring**

$\phi$ at points on the axis

$$r = \sqrt{(z^2 + R^2)} \text{ for ALL SEGMENTS}$$

$$\phi(z) = \int \frac{k dQ}{\sqrt{z^2 + R^2}} = \frac{k}{\sqrt{z^2 + R^2}} \int dQ = \frac{kQ}{\sqrt{z^2 + R^2}}$$

- Look at the potential far away from the ring
- From here, get to the uniformly charged disk (in book)
Potential of a dipole
Potential “far away” from a dipole

\[ \phi(r, \theta) \rightarrow \frac{p \cos \theta}{4 \pi \epsilon_0 r^2} \]
Electric field “far away” from a dipole

\[
\phi(r, \theta) \to \frac{p \cos \theta}{4\pi\varepsilon_0 r^2}
\]

\[
E(r, \theta) = \frac{p}{4\pi\varepsilon_0 r^3} (2 \cos \theta \hat{r} + \sin \theta \hat{\theta})
\]
(note that the points in this plot are not “far away”)
The smiling face of Fig. 24-44 consists of three items:

1. a thin rod of charge $-3.0 \mu C$ that forms a full circle of radius 6.0 cm;
2. a second thin rod of charge $2.0 \mu C$ that forms a circular arc of radius 4.0 cm, subtending an angle of 90° about the center of the full circle;
3. an electric dipole with a dipole moment that is perpendicular to a radial line and has magnitude $1.28 \times 10^{-21} \text{ C} \cdot \text{m}$.

What is the net electric potential at the center?
Charges and Potential

Given the potential \( \varphi(\vec{r}) \)
get the field from the potential
get the charge from the field

Combine to get the charge directly from \( \varphi(\vec{r}) \)

\[
\nabla \cdot E = -\nabla \cdot \nabla \varphi = -\left( \frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2} + \frac{\partial^2 \varphi}{\partial z^2} \right)
\]

Poisson equation (from Gauss’ law)

\[
\nabla^2 \varphi = -\frac{\rho}{\epsilon_0}
\]
Charges and Potential

Another way to get $\varphi(\vec{r})$ from $\rho(\vec{r})$

Poisson equation

in charge-free regions, Laplace equation

$\nabla^2 \phi = 0$

2$^{nd}$ order diff equation

“initial conditions” -> “boundary conditions”
Rules for charges and electric fields in electrostatic systems with CONDUCTORS
Rules for charges and electric fields in electrostatic systems with CONDUCTORS

Fact #1: electric field is ZERO at all points inside a conductor

A nonzero field would result in a nonzero force on the free charges at that point and they would move

They are not moving (electrostatics) so there is no force acting on them and the field must be zero.
Rules for charges and electric fields in electrostatic systems with CONDUCTORS

Fact #1: electric field is ZERO at all points inside a conductor

Gauss’ law: electric fields determine charge distribution

\[ \nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0} \]

So fact #1 + Gauss’ Law =
Net charge is ZERO at all points inside a conductor
Rules for charges and electric fields in electrostatic systems with CONDUCTORS

Fact #1: electric field is ZERO at all points inside a conductor

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Net charge is ZERO at all points inside a conductor

How can this be if the conductor has a net charge?
Rules for charges and electric fields in electrostatic systems with CONDUCTORS

Net charge is ZERO at all points **INSIDE** a conductor

**BUT**

Charge can be nonzero at points on surface of a conductor

surface charge density \( \sigma(r) \) (charge/area)
Rules for charges and electric fields in electrostatic systems with CONDUCTORS

Net charge is ZERO at all points INSIDE a conductor
BUT
Charge can be nonzero at points on surface of a conductor

Surface charge density \( \sigma(r) \) (charge/area)

\[ \oint \sigma \, dA = Q \]
Rules for charges and electric fields in electrostatic systems with CONDUCTORS

Charge at the surface arranges itself so that the electric field inside is zero

Is the charge distribution on the spherical conductor uniform?
Rules for charges and electric fields in electrostatic systems with CONDUCTORS

Charge at the surface arranges itself so that the electric field inside is zero

Is the charge distribution on the spherical conductor uniform?
Rules for charges and electric fields in electrostatic systems with CONDUCTORS

Charge at the surface arranges itself so that the electric field inside is zero

Neutral spherical conductor with a concentric spherical cavity
Put a point charge at the center of the cavity
How is the charge arranged on surface of the conductor?
What is the field OUTSIDE the conductor?
Rules for charges and electric fields in electrostatic systems with CONDUCTORS

Charge at the surface arranges itself so that the electric field inside is zero

Neutral spherical conductor with a concentric spherical cavity
Put a point charge at the center of the cavity
How is the charge arranged on surface of the conductor?
What is the field OUTSIDE the conductor?

Suppose the spherical conductor has charge Q
Electric fields are zero inside conductors
All excess charge is on the surface

What can we say about the field at the surface of a conductor?

It is perpendicular to the surface
So the surface is an equipotential
All interior points are at the same potential as the surface
(field inside is zero)

Magnitude of the field at the surface is $\sigma/\varepsilon_0$
(Use Gauss’s Law!)
Rules for charges, fields & potential in electrostatic systems with conductors

Charges go to the surface (includes exterior and interior)
No net charge at any point in the interior
Field in the interior is zero
Field at surface is perpendicular to the surface
Potential is the same at all points in the conductor
Field at surface is $\sigma/\varepsilon_0$

True for an isolated conductor
True for a conductor in the presence of other charges, conductors
Rule for conductors with cavities:
IF the cavity is empty then the field is zero
IF the cavity contains charges, then field is not zero in the cavity
But the net charge on the surface of the cavity = -net charge inside