TODAY

wave equation for E and B fields
Wave solutions of Maxwell’s equations
Energy and momentum in EM waves
Polarization of EM waves
Electromagnetism

\[ \nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0} \quad \nabla \cdot \mathbf{B} = 0 \]

\[ \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad \nabla \times \mathbf{B} = \mu_0 \mathbf{j} + \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}. \]

Maxwell’s equations: fields, charge, current

\[ \mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B}). \]
Electric and magnetic fields in empty space

\[ \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad \nabla \times \vec{B} = \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t} \]

\[ \nabla \cdot \vec{E} = 0 \quad \nabla \cdot \vec{B} = 0 \]

Yes, \( \vec{E} = \vec{B} = 0 \) is a solution BUT there are also solutions with nonzero \( \vec{E} \) and \( \vec{B} \)
Combine the Maxwell equations with \( \rho = j = 0 \) (empty space) to show

\[
\frac{d^2 \vec{E}}{dx^2} + \frac{d^2 \vec{E}}{dy^2} + \frac{d^2 \vec{E}}{dz^2} - \mu_0 \varepsilon_0 \frac{d^2 \vec{E}}{dt^2} = 0 \quad \frac{d^2 \vec{B}}{dx^2} + \frac{d^2 \vec{B}}{dy^2} + \frac{d^2 \vec{B}}{dz^2} - \mu_0 \varepsilon_0 \frac{d^2 \vec{B}}{dt^2} = 0
\]

compare

\[
\frac{\partial^2 \mathbf{y}}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 \mathbf{y}}{\partial t^2}
\]

(wave equation).

E and B obey wave equations!
Wave speed = \((\mu_0 \varepsilon_0)^{-1/2}\) = 2.992 \times 10^8 \text{ m/s}

Compare measured speed of light
(in 1850, using mirrors 8 km apart)
\(c = 3.0 \times 10^8 \text{ m/s}

Maxwell’s aha moment!
Electromagnetic spectrum: Wavelength $\lambda = c/f$
Heinrich Hertz 1880: confirmed the existence of electromagnetic waves beyond the visible range

Fig. 33-3  An arrangement for generating a traveling electromagnetic wave in the shortwave radio region of the spectrum: an LC oscillator produces a sinusoidal current in the antenna, which generates the wave. P is a distant point at which a detector can monitor the wave traveling past it.
One solution of these wave equations

\[ \frac{d^2 \vec{E}}{dx^2} + \frac{d^2 \vec{E}}{dy^2} + \frac{d^2 \vec{E}}{dz^2} - \frac{1}{c^2} \frac{d^2 \vec{E}}{dt^2} = 0 \]

\[ \frac{d^2 \vec{B}}{dx^2} + \frac{d^2 \vec{B}}{dy^2} + \frac{d^2 \vec{B}}{dz^2} - \frac{1}{c^2} \frac{d^2 \vec{B}}{dt^2} = 0 \]

\[ E_y(x, y, z, t) = E_m \sin(kx - \omega t); \quad E_x = E_z = 0 \]

\[ B_x(x, y, z, t) = B_m \sin(k'x - \omega' t); \quad B_y = B_z = 0 \]

With \( \omega/k = \omega'/k' = c \)
\[
E_y(x, y, z, t) = E_m \sin(kx - \omega t); \ E_x = E_z = 0 \]

\[
B_x(x, y, z, t) = B_m \sin(k'x - \omega' t); \ B_y = B_z = 0 \]

With \( \frac{\omega}{k} = \frac{\omega'}{k'} = c \)

\[
\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad \nabla \times \vec{B} = \frac{1}{c^2} \frac{\partial \vec{E}}{\partial t} \\
\nabla \cdot \vec{E} = 0 \quad \nabla \cdot \vec{B} = 0
\]
\[ E_y(x, y, z, t) = E_m \sin(kx - \omega t); \quad E_x = E_z = 0 \]

\[ B_x(x, y, z, t) = B_m \sin(k'x - \omega' t); \quad B_y = B_z = 0 \]

With \( \omega/k = \omega'/k' = c \)

\[ \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad \nabla \times \vec{B} = \frac{1}{c^2} \frac{\partial \vec{E}}{\partial t} \]

\[ \nabla \cdot \vec{E} = 0 \quad \nabla \cdot \vec{B} = 0 \]

NOT a solution of Maxwell’s equations!
Simplest example of fields that solve Maxwell’s equations: “Linearly polarized plane wave”

Must have \( \text{div } E = 0 \)
\[ E_y(x, y, z, t) = E_m \sin(kx - \omega t) \]
\( E_x = E_z = 0 \)
\( \omega/k = c \)
\[ E_y(x, y, z, t) = E_m \sin(kx - \omega t); \ E_x = E_z = 0 \]

With \( \omega / k = \omega' / k' = c \)

\[ \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \]

\[ \vec{B}(x, y, z, t) = \hat{k}E_m \frac{k}{\omega} \sin(kx - \omega t) \]

\[ B_z(x, y, z, t) = B_m \sin(kx - \omega t) \]

with \( B_m = E_m / c \)
\[ E_y(x, y, z, t) = E_m \sin(kx - \omega t); \ E_x = E_z = 0 \]

\[ B_z(x, y, z, t) = B_m \sin(kx - \omega t); \ B_m = E_m/c, \ B_x = B_y = 0 \]

with \( \omega/k = c \)

\[ \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad \nabla \times \vec{B} = \frac{1}{c^2} \frac{\partial \vec{E}}{\partial t} \]

\[ \nabla \cdot \vec{E} = 0 \quad \nabla \cdot \vec{B} = 0 \]

NOW a solution of Maxwell’s equations!
Simplest example of fields that solve Maxwell’s equations: “Linearly polarized plane wave”

\[ E_y(x, y, z, t) = E_m \sin(kx - \omega t) \]
\[ E_x = E_z = 0 \]
\[ \omega/k = c \]

then

\[ B_z(x, y, z, t) = (E_m/c)\sin(kx - \omega t) \]
Electromagnetic wave traveling in +x direction

E and B fields are perpendicular to direction of travel
E and B are perpendicular to each other
E/B = c

Changing B produces E
Changing E produces B
self-sustaining
1. If the magnetic field of a light wave oscillates parallel to a $y$ axis and is given by $B_y = B_m \sin(kz - \omega t)$, (a) in what direction does the wave travel and (b) parallel to which axis does the associated electric field oscillate?
Simple “linearly polarized” electromagnetic wave

Direction of propagation
Wavelength (or frequency)
Direction of the oscillating E field = “polarization”

\[ E_y(x, y, z, t) = E_m \sin(kx - \omega t) \]
\[ B_z(x, y, z, t) = B_m \sin(kx - \omega t) \]

Vertically polarized light headed toward you—the electric fields are all vertical.
4. Figure 33-28 shows the electric and magnetic fields of an electromagnetic wave at a certain instant. Is the wave traveling into the page or out of it?
Electromagnetic waves transport energy (radiation from the sun) measure using “intensity” = (energy/area)/time

\[ I = \frac{\text{power}}{\text{area}} = \frac{P_s}{4\pi r^2}, \]

where \( P_s \) is the source power.
Assume (unrealistically) that a TV station acts as a point source broadcasting isotropically at 1.0 MW. What is the intensity of the transmitted signal reaching Proxima Centauri, the star nearest our solar system, 4.3 ly away? (An alien civilization at that distance might be able to watch *X Files.*) A light-year (ly) is the distance light travels in one year.
EM wave transports energy

Energy density associated with E field = \( \frac{\varepsilon_0}{2} |\vec{E}(\vec{r})|^2 \)

Energy density associated with B field = \( \frac{1}{2\mu_0} |\vec{B}(\vec{r})|^2 \)

\( E_y(x, y, z, t) = E_m \sin(kx - \omega t) \)

\( B_z(x, y, z, t) = B_m \sin(kx - \omega t) \)

Energy/area in one wavelength =

\[ \lambda \left( \frac{\varepsilon_0 E_m^2}{4} + \frac{B_m^2}{4\mu_0} \right) \]

= \( \lambda \left( \frac{\varepsilon_0 E_m^2}{2} \right) \)

(energy associated with E field = energy associated with B field)

Rate at which energy passes through unit area perpendicular to direction of propagation is (energy/area)/time

\[ c\varepsilon_0 E_m^2 / 2 = c\varepsilon_0 E_{rms}^2 \]
Rate at which energy passes through unit area perpendicular to direction of propagation is

$$c \varepsilon_0 E_m^2 / 2 = c \varepsilon_0 E_{rms}^2 = I$$

You can think of an energy current analogous to a charge current = energy/time passing through a surface

“Energy current density” $S$ for a general traveling wave should satisfy continuity equation where $U$ is energy density

$$\nabla \cdot \vec{S} = -\frac{\partial U(\vec{r}, t)}{\partial t}$$

For em waves

$$\vec{S}(\vec{r}, t) = (\vec{E} \times \vec{B}) / \mu_0$$

Confirm it gives the right rate for our simple plane wave (next slide)
33.5: Energy Transport and the Poynting Vector:

The direction of the Poynting vector $\vec{S}$ of an electromagnetic wave at any point gives the wave’s direction of travel and the direction of energy transport at that point.

$$\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B} \quad \text{(Poynting vector).}$$

Linearly polarized wave $E$ is perpendicular to $B$ and to direction of propagation.

$$S = \frac{1}{\mu_0} EB, \quad \Rightarrow \quad S = \frac{1}{c\mu_0} E^2$$

$$I = S_{\text{avg}} = \left( \frac{\text{energy/time}}{\text{area}} \right)_{\text{avg}} = \left( \frac{\text{power}}{\text{area}} \right)_{\text{avg}} = \frac{1}{c\mu_0} [E^2]_{\text{avg}} = \frac{1}{c\mu_0} \left[ E_m^2 \sin^2(kx - \omega t) \right]_{\text{avg}}.$$}

$$E_{\text{rms}} = \frac{E_m}{\sqrt{2}}.$$}

$$I = \frac{1}{c\mu_0} E_{\text{rms}}^2 \Rightarrow \quad I = \frac{1}{2} c\varepsilon_0 E_m^2 = \text{speed x energy per volume}$$
15. An airplane flying at a distance of 10 km from a radio transmitter receives a signal of intensity $10 \, \mu \text{W/m}^2$. What is the amplitude of the (a) electric and (b) magnetic component of the signal at the airplane? (c) If the transmitter radiates uniformly over a hemisphere, what is the transmission power?
EM wave also transports **momentum**
(direction of momentum is parallel to direction of propagation)

Rate at which this momentum passes through unit area perpendicular to direction of propagation is $I/c$

$\frac{\text{momentum/time}}{\text{area}} = \frac{\text{force/area}}{\text{area}} = \text{pressure}$

“radiation pressure” $P = I/c$

If wave hits a perfectly reflecting surface, then $P = 2I/c$
Simple “linearly polarized” electromagnetic wave

Direction of propagation
Wavelength (or frequency)
Amplitude of the oscillating E field: intensity = power/area
Direction of the oscillating E field = “polarization”

\[ E_y(x, y, z, t) = E_m \sin(kx - \omega t) \]
\[ B_z(x, y, z, t) = B_m \sin(kx - \omega t) \]
“Polarizing sheet”

An electric field component parallel to the polarizing direction is passed (*transmitted*) by a polarizing sheet; a component perpendicular to it is absorbed.

- Transmitted
- Absorbed
- ????
An electric field component parallel to the polarizing direction is passed (transmitted) by a polarizing sheet; a component perpendicular to it is absorbed.

Component along polarizing direction is transmitted: $E \cos \phi$

Effect is to rotate the polarization and reduce the intensity $I$ proportional to $E_m^2$

So transmitted intensity $I = I_0 (\cos \phi)^2$

If the angle is 45 degrees, the intensity decreases by a factor of 2
“Unpolarized” light

Random mixture of directions

e.g. light bulb

One polarizing sheet:
Unpolarized light of intensity $I$ -> linearly polarized; $I = I_0/2$

Stack another polarizing sheet on top:
Rotate: no difference ---- completely black
Two polarizing sheets are stacked with directions at right angles and placed on the projector, and transmit no light. A third sheet with direction at 45 degrees to each is inserted between the two sheets. What is then true about the intensity $I$ of the light transmitted by the 3-sheet stack?

a) No light is transmitted
b) $I = I_0$
c) $I = I_0/2$
d) $I = I_0/4$
e) $I = I_0/8$

ANSWER WITH THE DEMO!