Exam 2 return

Grade distribution
mean & median 63
94+   3
80-90 4
70-80 3
60-70 4
<60   9
MAXWELL’S EQUATIONS
FOR TIME-INDEPENDENT CHARGE AND CURRENT DISTRIBUTIONS

\[ \nabla \times \vec{E} = 0 \]
\[ \nabla \cdot \vec{E} = \rho / \varepsilon_0 \]
\[ \nabla \times \vec{B} = \mu_0 \vec{J} \]
\[ \nabla \cdot \vec{B} = 0 \]

\( \rho \) determines \( \vec{E} \)

\( \rho \) determines \( \vec{E} \)

\( \vec{J} \) determines \( \vec{B} \)

\[ d\vec{E} = k \frac{dq}{r^2} \hat{r} \]

Coulomb’s law

\[ d\vec{B} = \frac{\mu_0}{4\pi} \frac{i d\vec{s} \times \hat{t}}{r^2} \]

Biot-Savart law

Electric and magnetic fields are time-independent
Waving a bar magnet around produced an emf in a coil

\[ \oint E \cdot d\vec{s} = - \frac{d\Phi_B}{dt} \quad \text{(Faraday’s law).} \]

\[ \vec{\nabla} \times \vec{E} = - \frac{\partial \vec{B}}{\partial t} \]

A changing magnetic field produces an electric field

This is in addition to the electric field produced by charges.
A changing ELECTRIC field produce a MAGNETIC field? YES!
This is in addition to magnetic fields produced by currents:
“displacement current” density $\varepsilon_0 \frac{dE}{dt}$ : Maxwell-Ampere law

$$\nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

$$\nabla \times \vec{B} = \mu_0 (\vec{J} + \varepsilon_0 \frac{\partial \vec{E}}{\partial t})$$
How to get a changing electric field?

Charging or discharging capacitor

Changing current in wire

Charge accumulation at end of a wire
Before charging, there is no magnetic field.

During charging, magnetic field is created by both the real and fictional currents.

During charging, the right-hand rule works for both the real and fictional currents.

After charging, there is no magnetic field.
17. A silver wire has resistivity $\rho = 1.62 \times 10^{-8} \, \Omega \cdot \text{m}$ and a cross-sectional area of $5.00 \, \text{mm}^2$. The current in the wire is uniform and changing at the rate of $2000 \, \text{A/s}$ when the current is $100 \, \text{A}$. (a) What is the magnitude of the (uniform) electric field in the wire when the current in the wire is $100 \, \text{A}$? (b) What is the displacement current in the wire at that time? (c) What is the ratio of the magnitude of the magnetic field due to the displacement current to that due to the current at a distance $r$ from the wire?
A silver wire has resistivity $\rho = 1.62 \times 10^{-8} \Omega \cdot m$ and a cross-sectional area of 5.00 mm$^2$. The current in the wire is uniform and changing at the rate of 2000 A/s when the current is 100 A. (a) What is the magnitude of the (uniform) electric field in the wire when the current in the wire is 100 A? (b) What is the displacement current in the wire at that time? (c) What is the ratio of the magnitude of the magnetic field due to the displacement current to that due to the current at a distance $r$ from the wire?

\[
E = \rho j = \rho I/A
\]
\[
I_D = \epsilon_0 dE/dt = \epsilon_0 \rho (dI/dt)/A
\]
\[
B_D/B_c = I_D/I
\]

(a) 0.324 V/m; (b) $2.87 \times 10^{-16}$ A; (c) $2.87 \times 10^{-18}$
MAXWELL’S EQUATIONS
FOR TIME-DEPENDENT CHARGE AND CURRENT DISTRIBUTIONS

\[ \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad \nabla \times \vec{B} = \mu_0 (\vec{J} + \varepsilon_0 \frac{\partial \vec{E}}{\partial t}) \]

\[ \nabla \cdot \vec{E} = \rho / \varepsilon_0 \quad \nabla \cdot \vec{B} = 0 \]

\( \rho \) and \( dB/dt \) determine \( \vec{E} \)
\( J \) and \( dE/dt \) determine \( \vec{B} \)
Electromagnetism

\[ \nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0} \]
\[ \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \]
\[ \nabla \cdot \mathbf{B} = 0 \]
\[ \nabla \times \mathbf{B} = \mu_0 \mathbf{j} + \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}. \]

Maxwell’s equations: fields, charge, current

\[ \mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B}). \]
And now for something completely different...
Speculations about the nature of light

Light propagates through space
Stream of particles OR a wave
(Newton vs Huygens)

Young double slit interference -> wave
Visible light has wavelengths 400-700 nm

Speed of light is $3 \times 10^8$ m/s
Danish astronomer Ole Romer – watching Io go around Jupiter – delay in observation depending how far away Jupiter was
9th November 1676
QUICK REVIEW OF WAVES
Wave Motion:

A wave is a disturbance that transports energy away from its source.

“Mechanical waves” propagate in a medium (a string, air, a solid…)
Wave may propagate over a large distance.
But particles of the medium move in a much more localized area.

https://phet.colorado.edu/sims/html/wave-on-a-string/latest/wave-on-a-string_en.html
In a **transverse** wave, the displacement of every oscillating element along the wave is **perpendicular** to the direction of travel of the wave.
\[ y(x,t) = y_m \sin (kx - \omega t) \text{ or } y(x,t) = y_m \sin (kx + \omega t) \]

\[ y(x,t) = y_m \sin (kx - \omega t) \]

To keep \( kx - \omega t \) constant, \( t \to t + \Delta t \) means \( x \to x + (\omega/k) \Delta t \)

wave traveling to the right with speed \( v = \Delta x/\Delta t = \omega/k \)

\[ y(x,t) = y_m \sin (kx + \omega t) \]

wave traveling to the left
In one period, the pattern returns to itself
Crest has moved one wavelength to the right

Wavespeed $v = \text{wavelength/period} = \lambda/T$
Wavespeed $v = (2\pi/k)/(2\pi/\omega) = \omega/k$
QUICK REVIEW OF WAVES

Waves on a string: \( y(x,t) = y_m \sin(kx-\omega t) \)

Wave speed \( v = \omega/k \), frequency \( f = \omega/2\pi \), wavelength \( \lambda = 2\pi/k \)

wave crests, pulse move down the string at speed \( v \)
\( v \) does not depend on the wavelength
Slinky demo
Measure dependence of wave speed on tension

PHET demo “experiment”
Measure dependence of wave speed on frequency
PHET demo “experiment”
Measure the dependence of wave speed on frequency
https://phet.colorado.edu/sims/html/wave-on-a-string/latest/wave-on-a-string_en.html

For a given string,
WAVE SPEED IS INDEPENDENT OF FREQUENCY

-> a pulse of any shape moves along the string at speed $v$
(Fourier decomposition)
QUICK REVIEW OF WAVES

Waves on a string: \( y(x,t) = y_m \sin(kx - \omega t) \)

Wave speed \( v = \frac{\omega}{k} \), frequency \( f = \frac{\omega}{2\pi} \), wavelength \( \lambda = \frac{2\pi}{k} \)

wave crests, pulse move down the string at speed \( v \)
\( v \) does not depend on the wavelength

Waves transport energy
- kinetic energy of the moving string element
- potential energy to stretch the string away from horizontal
The string displacements satisfy the wave equation

\[ \frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 y}{\partial t^2} \]  

(wave equation).

\( v \) is the wave speed

For the string, the coefficient \( 1/v^2 \) is \( \mu/\tau \)
\( \mu \) is the mass density
\( \tau \) is the tension in the string
Electromagnetism

\[ \oint \vec{E} \cdot d\vec{A} = q_{\text{enc}}/\varepsilon_0 \]
\[ \oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt} \]
\[ \oint \vec{B} \cdot d\vec{A} = 0 \]
\[ \oint \vec{B} \cdot d\vec{s} = \mu_0\varepsilon_0 \frac{d\Phi_E}{dt} + \mu_0i_{\text{enc}} \]

Maxwell’s equations: fields, charge, current

\[ \mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B}). \]
What can we say about E and B when the charge and current distributions are zero (empty space)?
Electric and magnetic fields in empty space

\[ \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \]
\[ \nabla \times \vec{B} = \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t} \]

\[ \nabla \cdot \vec{E} = 0 \]
\[ \nabla \cdot \vec{B} = 0 \]

Yes, \( E=B=0 \) is a solution BUT there are also solutions with nonzero \( E \) and \( B \)
Key vector identity

\[ \vec{\nabla} \times (\vec{\nabla} \times \vec{A}) = \vec{\nabla}(\vec{\nabla} \cdot \vec{A}) - \nabla^2 \vec{A} \]
Let’s use the vector identity

\[ \nabla \times (\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} ) \]

\[ \nabla (\nabla \cdot \vec{E}) - \nabla^2 \vec{E} = -\mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2} \]

\[ \nabla^2 \vec{E} - \mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2} = 0 \]
Combine the Maxwell equations with ρ = j = 0 (empty space) to show

\[
\begin{align*}
\frac{d^2 \vec{E}}{dx^2} + \frac{d^2 \vec{E}}{dy^2} + \frac{d^2 \vec{E}}{dz^2} - \mu_0 \varepsilon_0 \frac{d^2 \vec{E}}{dt^2} &= 0 \\
\frac{d^2 \vec{B}}{dx^2} + \frac{d^2 \vec{B}}{dy^2} + \frac{d^2 \vec{B}}{dz^2} - \mu_0 \varepsilon_0 \frac{d^2 \vec{B}}{dt^2} &= 0
\end{align*}
\]

**WAVE EQUATIONS**

Wave solutions for E(r,t) and B(r,t)

\[\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}\] connects E(r,t) and B(r,t)
Combine the Maxwell equations with $\rho = j = 0$ (empty space) to show

$$\frac{d^2 \vec{E}}{dx^2} + \frac{d^2 \vec{E}}{dy^2} + \frac{d^2 \vec{E}}{dz^2} - \mu_0 \varepsilon_0 \frac{d^2 \vec{E}}{dt^2} = 0 \quad \frac{d^2 \vec{B}}{dx^2} + \frac{d^2 \vec{B}}{dy^2} + \frac{d^2 \vec{B}}{dz^2} - \mu_0 \varepsilon_0 \frac{d^2 \vec{B}}{dt^2} = 0$$

compare

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{\nu^2} \frac{\partial^2 y}{\partial t^2} \quad \text{(wave equation).}$$

E and B obey wave equations!
What is the wave speed?
Combine the Maxwell equations with $\rho = j = 0$ (empty space) to show

$$\frac{d^2 \vec{E}}{dx^2} + \frac{d^2 \vec{E}}{dy^2} + \frac{d^2 \vec{E}}{dz^2} - \mu_0 \varepsilon_0 \frac{d^2 \vec{E}}{dt^2} = 0 \quad \frac{d^2 \vec{B}}{dx^2} + \frac{d^2 \vec{B}}{dy^2} + \frac{d^2 \vec{B}}{dz^2} - \mu_0 \varepsilon_0 \frac{d^2 \vec{B}}{dt^2} = 0$$

compare

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 y}{\partial t^2} \quad \text{(wave equation)}.$$ 

E and B obey wave equations!

Wave speed $= (\mu_0 \varepsilon_0)^{-1/2} = 2.992 \times 10^8 \text{ m/s}$

Compare measured speed of light
(in 1850, using mirrors 8 km apart)

c $= 3.0 \times 10^8 \text{ m/s}$

Maxwell’s aha moment!
Electromagnetic spectrum: Wavelength $\lambda = \frac{c}{f}$