next HW due Wed April 1
next exam: WED APRIL 15
(format under discussion)

class web site
http://www.physics.rutgers.edu/ugrad/272
Recap: for long straight wire, if curve encloses wire

\[ \int_C \vec{B} \cdot d\vec{s} = \mu_0 i \]

if curve does not enclose wire

\[ \int_C \vec{B} \cdot d\vec{s} = 0 \]

This is a special case of a general relation between current distribution and field
General relation between the current distribution and the magnetic field, starting with the wire ("Ampere's law")

\[ \int_C \vec{B} \cdot d\vec{s} = \mu_0 i_{enc} \]

The left hand side is a line integral around the closed curve C.

On the right hand side we see \( \mu_0 \), which is a fundamental constant, and \( i_{enc} \), which is the current passing through the closed curve C (the flux of \( j \) through a surface spanned by C).
\( i_{\text{enc}} \) Positive or negative?

Take the loop and stretch a surface across it.

Any current that passes through the surface is included in \( i_{\text{enc}} \).

The positive direction is defined by a right-hand rule.

This is how to assign a sign to a current used in Ampere's law.

Fig. 29-12 A right-hand rule for Ampere’s law, to determine the signs for currents encircled by an Amperian loop. The situation is that of Fig. 29-11.
If your loop direction is clockwise and the current in the central wire is out of the page, what is the sign of the enclosed current?
(a) Positive
(b) Negative
(c) Zero
If your loop direction is clockwise and the current in the central wire is out of the page, what is the sign of the enclosed current?
(a) Positive
(b) Negative
(c) Zero

This is how to assign a sign to a current used in Ampere's law.
Ampere’s law

Verify Ampere’s law for a long straight wire with current $I$

if curve encloses wire, showed before that

$$\int_C \vec{B} \cdot d\vec{s} = \mu_0 i_{\text{enc}}$$

$i_{\text{enc}} = i$ so Ampere’s law is verified

if curve does not enclose wire, showed before that

$$\int_C \vec{B} \cdot d\vec{s} = 0$$

$i_{\text{enc}} = 0$ so Ampere’s law is verified
USE Ampere’s law for easy computation of the field of a long straight wire with current $i$

Symmetry analysis will give you the fact that the B field is tangent to a circle centered on the wire and that the magnitude depends only on the distance to the wire.
Ampere’s law

\[ \oint \overrightarrow{B} \cdot d\overrightarrow{s} = \mu_0 i_{\text{enc}} \]

Compute the field of the long straight wire

\[ 2\pi r B(r) = \mu_0 i \]
Here’s how the symmetry argument works:

Need two facts
If I changes sign, B changes sign (from Biot-Savart)
\[ F = q \vec{v} \times \vec{B} \]

- Reflection in plane normal to wire containing the particle
- I changes sign so B changes sign
- \( \vec{v} \) does not change
- So \( F \) changes sign

So \( F \) must be normal to the reflection plane and therefore
- \( F \) is parallel to the wire
- If B had a nonzero component parallel to the wire, \( F \) would have a nonzero component out of the screen, therefore

**B has no component parallel to the wire**

- Rotation around dashed line axis
- I changes sign so B changes sign
- \( \vec{v} \) changes sign
- So \( F \) does not change sign

So \( F \) must be pointing in the radial direction
- If B had a nonzero radial component, \( F \) would have a nonzero component out of the screen, therefore

**B has no component in the radial direction**

**THEREFORE B MUST BE INTO/OUT OF THE SCREEN**
USE Ampere's law for easy computation of the field INSIDE a long straight wire with current \( i \) and uniform current density \( j = \frac{i}{\pi R^2} \)

Only the current encircled by the loop is used in Ampere's law.

\[
\oint \mathbf{B} \cdot d\mathbf{s} = B \oint ds = B(2\pi r).
\]

\[
i_{\text{enc}} = i \frac{\pi r^2}{\pi R^2}.
\]

\[
B(2\pi r) = \mu_0 i \frac{\pi r^2}{\pi R^2}
\]

\[
B = \left( \frac{\mu_0 i}{2\pi R^2} \right) r \quad \text{(inside straight wire)}.
\]
SHELL THEOREM FOR UNIFORM CURRENT DENSITY IN CYLINDER OR HOLLOW CYLINDER

Field is the same as if all current INSIDE is collapsed to a wire at the center.
Magnetic Field on axis of straight cylindrical coil ("solenoid")

Current I, n turns/length, radius b

Consider each turn as a ring with current I at z
Compute the field on the axis at \(z_0\)

\[
\frac{dB_z}{dz} = \frac{\mu_0 b^2 \text{Indz}}{2(b^2 + (z_0 - z)^2)^{3/2}}
\]

\[
B_z = \int_{z_1}^{z_2} dB_z = \int_{z_1}^{z_2} \frac{\mu_0 b^2 \text{Indz}}{2(b^2 + (z_0 - z)^2)^{3/2}} = \frac{\mu_0 b^2 \text{In}}{2} \int_{z_1-z_0}^{z_2-z_0} \frac{dz'}{(b^2 + z'^2)^{3/2}}
\]

\[
= \frac{\mu_0 b^2 \text{In}}{2} \left( \frac{z_2 - z_0}{b^2(b^2 + (z_2 - z_0)^2)^{1/2}} - \frac{z_1 - z_0}{b^2(b^2 + (z_1 - z_0)^2)^{1/2}} \right)
\]

Limit for infinite coil is \(\mu_0 \text{ln}\)
Ampere’s law provides an easier way to get the field of the solenoid.

At points outside the cylinder, far away from the ends, the fields from the rings tend to cancel. Field goes to zero in the limit that the length of the solenoid goes to infinity.
Extra topic #1
Ampere’s law: field of an isolated uniform sheet of current

Surface current density $\vec{K}$ : A/m

Symmetry tells us that the field is parallel to the sheet and perpendicular to the current, also that it is equal and opposite above and below the sheet
Extra topic #2: Magnetic Field of rotating charged object

6.49 A disk with radius R and surface charge density $\sigma$ spins with angular frequency $\omega$.
What is the magnetic field at the center?

Spinning disk = set of rings of current
Ring at $r$ with width $dr$ has current $I(r)$

\[ I(r) = \frac{\sigma 2\pi r dr}{2\pi / \omega} = \sigma \omega r dr \]

\[ dB = \frac{\mu_0 I(r)}{2r} = \frac{\mu_0 \sigma \omega dr}{2} \]

\[ B = \int_{0}^{R} dB = \frac{\mu_0 \sigma \omega R}{2} \]
\[ i_{enc} = \oint \mathbf{J} \cdot d\mathbf{a} \]

S is any surface that spans the loop C

C = circle, S = disk
Ampere’s law

\[ \oint B \cdot d\vec{s} = \mu_0 i_{enc} \]
Ampere’s law

\[ \oint \vec{B} \cdot d\vec{s} = \mu_0 i_{\text{enc}} \]

\[ \int_C \vec{B} \cdot d\vec{s} = \mu_0 \int_S \vec{J} \cdot d\vec{a} \]

\[ \int_C \vec{B} \cdot d\vec{s} = \int_S \left( \nabla \times \vec{B} \right) \cdot d\vec{a} \]

\[ \nabla \times \vec{B} = \mu_0 \vec{J} \]

Differential form of Ampere’s Law
General relation between the current distribution and the magnetic field, starting with the wire ("Ampere’s law")

\[ \nabla \times \vec{B}(\vec{r}) = \mu_0 j(\vec{r}) \]

\[ \int_C \vec{B} \cdot d\vec{s} = \mu_0 i_{enc} \]

The left hand side is a line integral around the closed curve C.

On the right hand side we see \( \mu_0 \), which is a fundamental constant, and \( i_{enc} \), which is the current passing through the closed curve C (the flux of j through a surface spanned by C).
Ampere’s law alone (that is, without additional information about the field) is not enough to determine the magnetic field. Lots of different vector fields have the same curl.

Let’s say we are given the current distribution $\vec{J}(\vec{r})$ and we have found a field $\vec{F}(\vec{r})$ such that

$$\nabla \times \vec{F}(\vec{r}) = \mu_0\vec{J}(\vec{r})$$

Is $\vec{F}(\vec{r})$ the magnetic field?
Ampere’s law alone (that is, without additional information about the field) is not enough to determine the magnetic field. Lots of different vector fields have the same curl.

Let’s say we are given the current distribution $\vec{J}(\vec{r})$ and we have found a field $\vec{F}(\vec{r})$ such that

$$\nabla \times \vec{F}(\vec{r}) = \mu_0 \vec{J}(\vec{r})$$

Is $\vec{F}(\vec{r})$ the magnetic field?
Ampere’s law alone (that is, without additional information about the field) is not enough to determine the magnetic field. Lots of different vector fields have the same curl.

Let’s say we are given the current distribution \( \vec{J}(\vec{r}) \) and we have found a field \( \vec{F}(\vec{r}) \) such that

\[
\nabla \times \vec{F}(\vec{r}) = \mu_0 \vec{J}(\vec{r})
\]

Is \( \vec{F}(\vec{r}) \) the magnetic field?

Not necessarily:

\[
\vec{F}'(\vec{r}) = \vec{F}(\vec{r}) + \nabla f(\vec{r})
\]

has the same curl as \( \vec{F}(\vec{r}) \) because of the vector identity \( \nabla \times \nabla f(\vec{r}) \)
Many different vector fields could have the same curl – how do we find the magnetic field from \( J(r) \)?

**NEED MORE INFORMATION** about \( B(r) \)
Many different vector fields could have the same curl – how do we find the magnetic field from $J(r)$?

NEED MORE INFORMATION about $B(r)$

All field lines form closed loops

So the flux of $B$ through a closed surface $S$ is ZERO

\[
\oint_S \vec{B} \cdot d\vec{a} = 0
\]

\[
\nabla \cdot \vec{B} = 0
\]
A vector field is uniquely determined by its divergence and curl, assuming it goes to zero at infinity (this result is called the Helmholtz theorem)

\[ \nabla \times \vec{E} = 0 \quad \nabla \times \vec{B} = \mu_0 \vec{J} \]

\[ \nabla \cdot \vec{E} = \rho / \varepsilon_0 \quad \nabla \cdot \vec{B} = 0 \]

\( \rho \) determines \( E \) \hspace{1cm} \( J \) determines \( B \)

MAXWELL’S EQUATIONS FOR TIME-INDEPENDENT CHARGE AND CURRENT DISTRIBUTION
NEXT
The vector potential and “gauge choice”
Derivation of Biot-Savart Law

[blackboard presentation]