BEFORE CLASS: play with magnetism demos
A real battery

The voltage difference between the terminals decreases as the current through the battery increases.

Behaves like an ideal battery $\mathcal{E}$ in series with internal resistor $r$. 
Analyze a circuit with a real battery
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To determine $\mathcal{E}$ and $r$: make two measurements of current

1. Open circuit voltage $= \mathcal{E}$
2. Short circuit current $r = \mathcal{E}/I_{sc}$

Ammeter should have "zero" resistance
Voltmeter should have "infinite" resistance

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To determine $\mathcal{E}$ and $r$: make two measurements of current

(1) Open circuit voltage $= \mathcal{E}$
(2) short circuit current $r = \mathcal{E}/I_{sc}$

Ammeter should have “zero” resistance
Multiloop circuits

Junction rule: net current into a junction is ZERO
Loop rule applies to every loop
Analyze a multiloop circuit

In Fig. 27-41, the ideal batteries have emfs $\mathcal{E}_1 = 10.0 \, \text{V}$ and $\mathcal{E}_2 = 0.500 \mathcal{E}_1$, and the resistances are each $4.00 \, \Omega$. What is the current in (a) resistance 2 and (b) resistance 3?

**USE LOOP CURRENTS!**
Automatically satisfy junction rule
Circuit networks: batteries and resistors

State of the circuit is given by voltage at each “point”
Current through each resistor is determined by voltage difference

Analyzing a complicated circuit
• look for resistors in series and parallel
First step in analyzing a circuit: reduce by replacing resistors in series and in parallel with effective resistors

Some complicated looking circuits can be reduced to a single loop
In Fig. 27-70, the ideal battery has emf $\mathcal{E} = 30.0$ V, and the resistances are $R_1 = R_2 = 14$ $\Omega$, $R_3 = R_4 = R_5 = 6.0$ $\Omega$, $R_6 = 2.0$ $\Omega$, and $R_7 = 1.5$ $\Omega$. What are currents (a) $i_2$, (b) $i_4$, (c) $i_1$, (d) $i_3$, and (e) $i_5$?
In Fig. 27-70, the ideal battery has emf $\varepsilon = 30.0$ V, and the resistances are $R_1 = R_2 = 14$ $\Omega$, $R_3 = R_4 = R_5 = 6.0$ $\Omega$, $R_6 = 2.0$ $\Omega$, and $R_7 = 1.5$ $\Omega$. What are currents (a) $i_2$, (b) $i_4$, (c) $i_1$, (d) $i_3$, and (e) $i_5$?

Note: Easy to see that $R_5$ and $R_6$ are in parallel. Also $R_1$ and $R_2$ are in parallel, as well as $R_3$ and $R_4$. 
Concept of equivalent circuit elements

“a two-terminal box in a circuit is completely equivalent, in its behavior in any other circuit to which it may be connected, to a single voltage source $\mathcal{E}_{eq}$ with internal resistance $R_{eq}$”

(that is, a battery with internal resistance)

Thenevin’s theorem
equivalent circuit elements: simple examples when there is no emf in the box

resistors in series \( R_{eq} = R_1 + R_2 \)

resistors in parallel

Put a voltage \( V \) across the terminals
Look at the current flowing in and out

\( \mathcal{E}_{eq} = 0 \) for these examples
Thenevin – a quicker way to compute Req

set all EMFs to zero and find Req for the resulting resistor network

Problem: find Thenevin equivalent for
Magnetic Field and Forces
A NEW TYPE OF FORCE

Two metal objects attract
If one is flipped over, they repel

NOT gravity: attractive & repulsive
NOT electric: acts through a metal sheet
Associated with certain materials: LODESTONE

Alignment of lodestone needles: Force comes from nearby lodestone

Lodestone needles align N-S in the absence of a nearby lodestone COMPASS (Earth = lodestone)
Bar magnet =
rod of lodestone material

Forces exerted at the two ends are OPPOSITE

One end is labeled N, the other S (north/south)
Like ends repel, opposites attract
Force is exerted on lodestone by MOVING CHARGES

DEMO: wire with electric current and a lodestone needle

No current = no force
Force is exerted on lodestone by MOVING CHARGES

DEMO: wire with electric current and a lodestone needle below

Turn on current = needle aligns perpendicular to wire
Force is exerted on lodestone by MOVING CHARGES

DEMO: wire with electric current and a lodestone needle above

Turn on current = needle aligns perpendicular to wire, opposite
Force is exerted by lodestone or bar magnet ON MOVING CHARGES

DEMO: cathode ray tube
Beam of electrons from back of device hits the screen and makes it glow
Bar magnet deflects the beam
Charged objects exert forces by creating a vector field called the electric field $\vec{E}(r)$ producing a force $\vec{F} = q\vec{E}(r)$

Lodestone and bar magnets exert forces by creating a vector field called the **magnetic field** $\vec{B}(r)$

Bar magnet field: (near ends)

Today: start with a given magnetic field and find the magnetic force

Next lecture: learn now the magnetic field is produced by a given distribution of currents (atomic-scale in lodestone)
The magnetic force on a charged particle at $\vec{r}$

**Magnitude:**
- Proportional to $B$
- Proportional to magnitude of charge $|q|$
- Proportional to speed $v$ \(\rightarrow\) zero force if not moving
- Depends on angle between $\vec{v}$ and $\vec{B}$
  - zero if $\vec{v}$ and $\vec{B}$ are parallel or antiparallel

Magnitude = $|q|\ v\ B\ \sin\ \phi$

Units: 1 N/(C m/s) = 1 T (Tesla)
The magnetic force on a charged particle at $\vec{r}$

Magnitude: $|q| \, v \, B \, \sin \phi$

**Direction:**
Perpendicular to direction of $\vec{v}$
Perpendicular to direction of $\vec{B}$
-> normal to the plane defined by $\vec{v}$ and $\vec{B}$

Two choices – use right-hand rule to choose
Thumb along $v$; fingers along $B$: **out of palm**

To put it all together: $\vec{F} = q\vec{v} \times \vec{B}$
Cross product of two vectors
i-clicker:

A proton is traveling in the negative y direction. It enters a magnetic field pointing in the positive z direction. In which direction is the force on the proton?

a) +x
b) +y
c) -z
d) +z
e) None of the above
i-clicker:
A proton is traveling in the negative y direction. It enters a magnetic field pointing in the positive z direction. In which direction is the force on the proton?

a) +x
b) +y
c) -z
d) +z
e) None of the above (-x)
A particle of charge +1.0C with velocity (in m/s) 
\[ \vec{v} = 2\hat{i} + 3\hat{j} \]
moves through the uniform magnetic field (in T) 
\[ \vec{B} = 3\hat{i} - 1.5\hat{j} \]

What is the magnitude of the magnetic force on the particle?

a) 12 N  
b) 6.0 N  
c) 4.5 N  
d) 1.5 N  
e) None of the above
A particle of charge +1.0C with velocity (in m/s) 
\[ \vec{v} = 2\hat{i} + 3\hat{j} \]
moves through the uniform magnetic field (in T) 
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What is the magnitude of the magnetic force on the particle?

\begin{itemize}
  \item [a)] 12 N
  \item [b)] 6.0 N
  \item [c)] 4.5 N
  \item [d)] 1.5 N
  \item [e)] None of the above
\end{itemize}
Force on a current carrying wire in a uniform magnetic field

B points out of the screen
Magnetic force is to the right
Charges are constrained to move along the wire ->
Magnetic force on the charges is transmitted to the wire
i-clicker:
Negative charges are moving up in the wire which is in a magnetic field pointing **into** the screen. In which direction is the force on the wire?

a) To your right
b) To your left
c) Up
d) Down
e) zero
i-clicker:
Negative charges are moving down in the wire which is in a magnetic field to the left. In which direction is the force on the wire?

a) To your right
b) To your left
c) Up
d) Down
e) Out of the screen
f) Into the screen
g) It’s zero
i-clicker:
Negative charges are moving down in the wire which is in a magnetic field to the left. In which direction is the force on the wire?

a) To your right
b) To your left
c) Up
d) Down
e) Out of the screen
f) Into the screen
g) It’s zero
Force on a current carrying wire in a uniform magnetic field

$B$ points out of the screen
Magnetic force is to the right
Force on the charges is transmitted to the wire

\[ F = \sum q v B = L A n q v B = i L B \]
Force on a current carrying wire in a uniform magnetic field

B points out of the screen
Magnetic force is to the right
Force on the charges is transmitted to the wire

\[ \vec{F}_B = i\vec{L} \times \vec{B} \]

\( \vec{L} \) is in the direction of the current

Electrons are the free charges here
What is the direction of the net force on the rectangular loop with a uniform B field pointing to the right?

a) To the right
b) To the left
c) Out of the screen
d) Into the screen
e) zero
What is the direction of the net force on the rectangular loop with a uniform B field pointing to the right?

a) To the right
b) To the left
c) Out of the screen
d) Into the screen
e) zero
The torque due to the tangential component of the force causes an angular acceleration around the rotation axis.

Torque along the rotation axis
= $F_t r$ (positive)
i-clicker:
What is the sign of the torque on the rectangular loop rotating on the fixed axis with a uniform \( \vec{B} \) field pointing to the right?

a) Positive (up along axis)
b) Negative (down along axis)
c) Zero
d) Just guessing
i-clicker:
What is the sign of the torque on the rectangular loop rotating on the fixed axis with a uniform B field pointing to the right?

a) Positive (up along axis)
b) Negative (down along axis)
c) Zero
d) Just guessing

Torque = 2 \((iBA)(b/2) = iB(ab) = iB\text{ area}\)
Fig. 28-18 The elements of an electric motor. A rectangular loop of wire, carrying a current and free to rotate about a fixed axis, is placed in a magnetic field. Magnetic forces on the wire produce a torque that rotates it. A commutator (not shown) reverses the direction of the current every half-revolution so that the torque always acts in the same direction.
Motion of a charged particle $q$ in a uniform field $B v_0$ along a field line
Motion of a charged particle $q$ in a uniform field $B$ 
$v_0$ perpendicular to the field 

\[ F = q v_0 B \] 
out of the screen 
perpendicular to the displacement 
Magnetic force does no work on the particle 
does not change its speed
Motion of a charged particle $q$ in a uniform field $B$ $v_0$ perpendicular to $B$ (out of the screen)
Motion of a charged particle $q$ in a uniform field $B$ $v_0$ perpendicular to $B$ (out of the screen)

$F = q v_0 B$ -- downward for $q > 0$
perpendicular to the displacement
Magnetic force does no work on the particle
does not change its speed
Motion of a charged particle in crossed $E$ and $B$ field

- $B$ out of the screen
- $E$ upward

Magnetic force downward
Electric force upward
If $E = vB$, net force is zero and particle will not change velocity
Motion of a charged particle $q$ in a uniform field $B$ $v_0$ perpendicular to the field

$B$ out of the screen

Initially $F = q v_0 B$ downward
Motion of a charged particle $q$ in a uniform field $B$ $v_0$ perpendicular to the field

$B$ out of the screen

Initially $F = q v_0 B$ downward
Motion of a charged particle $q$ in a uniform field $B$ $v_0$ perpendicular to the field

- Speed is constant (why?)
- Force is perpendicular to motion, constant magnitude
- Uniform circular motion

Radius of the circle: $qv_0B = mv_0^2/r$ so $r = mv_0/qB$
Motion of a charged particle $q$ in a uniform field $B$
$v_{0x}$ perpendicular to the field
$v_{0z}$ parallel to the field

B out of the screen