Physics 272: Advanced Honors Physics
Exam 2
April 15, 2019

70 minutes
7 questions, 75 points total
1 single-sided 8.5"x11" self-prepared formula sheet

Vector operators

Cartesian coordinates

\[ ds = dx \hat{x} + dy \hat{y} + dz \hat{z} \]

\[ \nabla = \hat{x} \frac{\partial}{\partial x} + \hat{y} \frac{\partial}{\partial y} + \hat{z} \frac{\partial}{\partial z} \]

\[ \nabla f = \frac{\partial f}{\partial x} \hat{x} + \frac{\partial f}{\partial y} \hat{y} + \frac{\partial f}{\partial z} \hat{z} \]

\[ \nabla \cdot A = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z} \]

\[ \nabla \times A = \left( \frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} \right) \hat{x} + \left( \frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x} \right) \hat{y} + \left( \frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} \right) \hat{z} \]

\[ \nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2} \]
1. Three short questions
(a) (5 pts) State the steady current condition. Does the current distribution \( \vec{j}(\vec{r}, t) = ax \hat{x} \) satisfy the steady current condition? Show your work.

(b) (5 pts) State the definition of resistivity (preferably in the form of an equation). Which, if any or more than one, of the following is a unit of resistivity?

\[
\frac{N_s}{C_m} \quad \frac{Ns m^2}{C^2} \quad \frac{V_m}{s} \quad \frac{C}{V m s} \quad \frac{N m^2}{C A}
\]

(c) (5 pts) The infinitesimal segment of length \( ds \) carrying current \( I \) in the \( \hat{j} \) direction is shown in the figure. What is its contribution to the magnetic field at points \( P_1, P_2 \) and \( P_3 \)?
2. A current $I$ flows counterclockwise in the square loop of side $a$ shown in the figure. This loop is centered at the origin and is partly in a region of nonzero magnetic field $\vec{B}$ perpendicular to the page, shown as the shaded area.
(a) (2 pts) If the field is uniform with $\vec{B}(x,y) = B_0 \hat{k}$, what is the magnetic force on the vertical segment CD?
(b) (4 pts) If the field is nonuniform with $\vec{B}(x,y) = B_0(x/a) \hat{k}$, what is the magnetic force on the vertical segment CD?
(c) (4 pts) If the field is nonuniform with $\vec{B}(x,y) = B_0(y/a)^2 \hat{k}$, what is the magnetic force on the vertical segment CD?
3. When two bulbs A and B are connected in parallel, bulb A is twice as bright as bulb B.
   (a) (5 pts) What is the resistance \( R_B \) of bulb B in terms of the resistance \( R_A \) of bulb A?
   (b) (5 pts) If the two bulbs are placed in the circuit shown in the figure with \( R = 2 \, R_A \),
   what is the brightness of bulb B in terms of the brightness of bulb A?
   **OPTIONAL** If the brightnesses of the two bulbs when placed in the circuit shown are
   the same, what is \( R \) in terms of \( R_A \)?
4. An infinitely long cylinder of radius 2a centered on the z axis has

\[ \tilde{j}(\vec{r}) = j_0((r/a) - (r/a)^2)\hat{k} \]

(a) (2 pts) Where inside the cylinder is \( \tilde{j}(\vec{r}) \) equal to zero?
(b) (6 pts) Find the magnetic field at all points on the positive x axis.
(c) (2 pts) Where, if anywhere, inside the cylinder is \( \vec{B}(\vec{r}) \) equal to zero?
5. (10 pts) What is the Thévenin equivalent of the network shown in the figure if all resistors have resistance 200Ω? Draw it and label it, being sure to show the points A and B.
6. (10 pts) A system contains two infinite straight current-carrying wires as shown in the figure. What is the magnetic field at the point P?
7. A system consists of two conducting spheres of equal radius $a$ separated by $R > 2a$ as in the figure. If I put a charge of 2.0 C on sphere 1 and zero charge on sphere 2, the potential on sphere 1 is 5.0 V and the potential on sphere 2 is 3.0 V.

(a) (4 pts) If I put 2.0 C on sphere 2 and zero charge on sphere 2, what is the potential on each sphere?

(b) (6 pts) Find the capacitance matrix for this system.