11-5 The Yo-Yo

A yo-yo is a physics lab that you can fit in your pocket. If a yo-yo rolls down its string for a distance \( h \), it loses potential energy in amount \( mgh \) but gains kinetic energy in both translational \( \left( \frac{1}{2} M v_{\text{com}}^2 \right) \) and rotational \( \left( \frac{1}{2} I_{\text{com}} \omega^2 \right) \) forms. As it climbs back up, it loses kinetic energy and regains potential energy.

In a modern yo-yo, the string is not tied to the axle but is looped around it. When the yo-yo “hits” the bottom of its string, an upward force on the axle from the string stops the descent. The yo-yo then spins, axle inside loop, with only rotational kinetic energy. The yo-yo keeps spinning (“sleeping”) until you “wake it” by jerking on the string, causing the string to catch on the axle and the yo-yo to climb back up. The rotational kinetic energy of the yo-yo at the bottom of its string (and thus the sleeping time) can be considerably increased by throwing the yo-yo downward so that it starts down the string with initial speeds \( v_{\text{com}} \) and \( \omega \) instead of rolling down from rest.

To find an expression for the linear acceleration \( a_{\text{com}} \) of a yo-yo rolling down a string, we could use Newton’s second law just as we did for the body rolling down a ramp in Fig. 11-8. The analysis is the same except for the following:

1. Instead of rolling down a ramp at angle \( \theta \) with the horizontal, the yo-yo rolls down a string at angle \( \theta = 90^\circ \) with the horizontal.
2. Instead of rolling on its outer surface at radius \( R \), the yo-yo rolls on an axle of radius \( R_0 \) (Fig. 11-9a).
3. Instead of being slowed by frictional force \( T_f \), the yo-yo is slowed by the force \( T \) on it from the string (Fig. 11-9b).

The analysis would again lead us to Eq. 11-10. Therefore, let us just change the notation in Eq. 11-10 and set \( \theta = 90^\circ \) to write the linear acceleration as

\[
a_{\text{com}} = -\frac{g}{1 + I_{\text{com}}/MR_0^2},
\]

(11-13)

where \( I_{\text{com}} \) is the yo-yo’s rotational inertia about its center and \( M \) is its mass. A yo-yo has the same downward acceleration when it is climbing back up.

11-6 Torque Revisited

In Chapter 10 we defined torque \( \tau \) for a rigid body that can rotate around a fixed axis, with each particle in the body forced to move in a path that is a circle cen-