First homework is due next TUESDAY 11:59PM – Sept 17th
Next spring Phys 228H is using HRW10 and Webassign
Yes to multiterm access

BEFORE CLASS: come down and play with the charged balloons – look for repulsive and attractive forces

Get your iclicker device or app ready to roll
Today we might try the Cloud version of iclicker: join the class Honors Analytical Physics II at icloud.com

Course web site
http://www.physics.rutgers.edu/~karin/227H
Specifying charge distribution
Electric field from a fixed charge distribution

Uniform electric field: forces and torques

Reformulation of Coulomb’s law = Gauss’ law
Electric flux
Specifying a fixed distribution of charges

Point charges $Q_i, \vec{r}_i$

Continuous charge distributions
Continuous charge distributions

Total charge Q uniformly spread over a wire of length L

Take a small piece of length d: what is the charge?

\[ Q \left( \frac{d}{L} \right) = \left( \frac{Q}{L} \right) d \]

Define

\[ \lambda = \frac{Q}{L} \]

“Line charge density” \( \lambda \): charge/length (C/m)
Continuous charge distributions

Total charge $Q$ uniformly spread over a surface of area $A$

Take a small piece of area $A_1$: what is the charge?
$Q \left( \frac{A_1}{A} \right) = \left( \frac{Q}{A} \right) A_1$

Define
$\sigma = \frac{Q}{A}$
“surface charge density” $\sigma$: charge/area (C/m$^2$)
Continuous charge distributions

Total charge $Q$ uniformly distributed through a 3D region of volume $V$

Take a small chunk of volume $V_1$: what is the charge?
$Q \left( \frac{V_1}{V} \right) = \left( \frac{Q}{V} \right) V_1$

Define
$\rho = \frac{Q}{V}$
“charge density” $\sigma$: charge/volume ($\text{C/m}^3$)
Electric field from a fixed distribution of charges

Point charges $Q_i$, $\vec{r}_i$
Sum the field from each point charge

$$\vec{E}(\vec{r}) = \sum_i \frac{k Q_i}{|\vec{r} - \vec{r}_i|^2} \frac{(\vec{r} - \vec{r}_i)}{|\vec{r} - \vec{r}_i|}$$

Continuous charge distributions

Divide charge distribution into teeny tiny pieces
Integrate the field from each piece in the charge distribution
Thin wire of length L has total charge Q distributed uniformly. Find \(E_x\) and \(E_y\) at any point on the x axis with \(x > L\).

**Linear charge density:** \(\lambda = Q/L\)

\[
dE_y = 0 \quad \text{so} \quad E_y = 0
\]

\[
dE_x = k \frac{dq}{r^2} = k \lambda \frac{ds}{(x-s)^2} \quad \text{integrate from} \ s=0 \ \text{to} \ s=L
\]

\[
E_x = k \lambda \frac{L}{(x(x-L))}
\]

**LINE INTEGRAL over curve** \(\vec{c}(s) = (s,0)\) with \(0 \leq s \leq L\)
The Electric Field due to a Continuous Charge: Example

A uniformly charged wire of total charge $Q$ is bent into semicircle of radius $R$. Find $E_x$ and $E_y$ at center

Charge density $\lambda = Q/\pi R$ so that element $ds$ of the wire has charge $dq = \lambda ds$

Parametrize curve by $\theta$
$ds = Rd\theta$

$E_y = 0$ (cancellation, symmetry)

$dE_x = \left( \frac{k dq}{R^2} \right) (\sin \theta)$
$\theta$ runs from 0 to $\pi$
A uniformly charged wire of total charge $Q$ is bent into semicircle of radius $R$. Find $E_x$ and $E_y$ at center.

Charge density $\lambda = Q / \pi R$ so that element $ds$ of the wire has charge $dq = \lambda ds$.

Parametrize curve by $\theta$.

$$ds = R d\theta$$

Integrate $dE_x = \left(\frac{k\lambda R d\theta}{R^2}\right) \sin \theta$ from $\theta = 0$ to $\theta = \pi$.

$E_y = 0$

$E_x = 2 \frac{k \lambda}{R}$ (to the right)
Electric field of uniformly charged ring on its central axis

For $z >> R$, looks like point charge $q$

$E_x = E_y = 0$ (symmetry)

$E_z = \frac{kqz}{(z^2 + R^2)^{3/2}}$

For $z >> R$, looks like point charge $q$
Electric field of uniformly charged spherical shell $Q, R$
Gravitational force exerted by a uniform shell of mass on a particle outside the shell

A uniform spherical shell of matter attracts a particle that is outside the shell as if all the shell’s mass were concentrated at its center.

on a particle inside the shell

A uniform shell of matter exerts no net gravitational force on a particle located inside it.
Force exerted by a uniform shell

on a particle outside the shell

A uniform spherical shell of matter attracts a particle that is outside the shell as if all the shell’s mass were concentrated at its center.

A shell of uniform charge attracts or repels a charged particle that is outside the shell as if all the shell’s charge were concentrated at its center.

on a particle inside the shell

A uniform shell of matter exerts no net gravitational force on a particle located inside it.

If a charged particle is located inside a shell of uniform charge, there is no net electrostatic force on the particle from the shell.

Electric field inside the shell: zero
Outside: the same as if there were point charge $Q$ at center
Electric field of a charge $Q < 0$
Electric field of a spherical shell with $Q < 0$

inside the shell: zero
Outside: the same as if there were point charge $Q$ at center
The uniform electric field $\vec{E}(\vec{r}) = \vec{E}$
Given a uniform electric field $\mathbf{E}$, how will charge $q$ move? Charge will feel force $\mathbf{F} = q\mathbf{E}$ (same force for all positions) $\mathbf{F} = ma$, so $a = \mathbf{F}/m = q\mathbf{E}/m$ (constant acceleration)

$$y(t) = \frac{1}{2}at^2$$
$$y(x) = \frac{1}{2}a\left(\frac{x}{v_{0x}}\right)^2$$
The charge on the green particle in the figure is
(a) Positive
(b) Negative
(c) Zero
(d) There is not enough information to determine its charge
(e) If I put any of the other answers I would be just guessing
The charge on the green particle in the figure is
(a) Positive
(b) Negative – force is up, opposite to electric field
(c) Zero
(d) There is not enough information to determine its charge
(e) If I put any of the other answers I would be just guessing
Force on dipole in uniform field
Force on dipole in uniform field
Force on dipole in uniform field
NET FORCE IS ALWAYS ZERO
TORQUE on dipole in uniform field
torque on dipole in uniform field
TORQUE on dipole in uniform field

2 \( QE(d/2) \) into the screen

2 \( QE(d \sin \theta /2) \) into the screen

zero
TORQUE on dipole in uniform field
Q d sinθ
turns dipole to align with the field

2 QE(d/2) into the screen
2 QE(d sinθ /2) into the screen

zero
Charge distribution determines electric field sum, line integral, surface integral, volume integral

--shouldn’t there be an easier way to prove the shell theorem?
--does the electric field determine the charge distribution?
--how do you find the charge distribution from the electric field?
Charge distribution determines electric field
sum, line integral, surface integral, volume integral

--shouldn’t there be an easier way to prove the shell theorem?
--does the electric field determine the charge distribution?
--how do you find the charge distribution from the electric field?

REFORMULATION OF COULOMB’S LAW

\[ \oint_S \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0} \]  GAUSS’ LAW
Chapter 23

Gauss’ Law
What is a surface?

A surface is a set of points in three-dimensional space satisfying one equation (and optionally some inequalities)

\{(x,y,z) \text{ such that } z = 1\} \text{ (plane)}

\{(x,y,z) \text{ such that } x^2+y^2+z^2 = 100\} \text{ (sphere)}

\{(x,y,z) \text{ such that } z = 1 \text{ and } x^2+y^2 \leq 100\} \text{ (disk)}

Open surface: has a boundary

Closed surface: has no boundary, has an inside and an outside
\[ \oint_S \vec{E} \cdot d\vec{A} = q_{\text{enc}} / \varepsilon_0 \]

What is the right hand side?

$q_{\text{enc}}$ is the NET charge inside the closed surface $S$

$\varepsilon_0 = 1/(4 \pi k)$ or $k = 1/(4 \pi \varepsilon_0)$

“permittivity of free space”
What is the left hand side?
“Electric flux” through closed surface $S$

\[ \oint_S \vec{E} \cdot d\vec{A} = q_{enc} / \varepsilon_0 \]