First homework is due TONIGHT 11:59PM – Sept 13th
Do the two pre-recitation problems before class
2nd homework is due next Tuesday 11:59PM

BEFORE CLASS: come down and play:
No electric field inside the metal cage!
• Radio inside cylindrical cage
• Person inside the flexible screen bag

Get your iclicker device or app ready to roll

Course web site
http://www.physics.rutgers.edu/~karin/227H
Charge distribution determines electric field
sum, line integral, surface integral, volume integral

--shouldn’t there be an easier way to prove the shell theorem?
--does the electric field determine the charge distribution?
--how do you find the charge distribution from the electric field?

REFORMULATION OF COULOMB’S LAW

\[ \oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{enc}}}{\varepsilon_0} \]

GAUSS’ LAW

electric flux through the closed surface S
Graphical approach to electric flux

Flux integral is proportional to
# of field lines out - # of field lines in
Graphical approach:

Flux integral is proportional to 
# of field lines out - # of field lines in

Especially useful for cases in which 
the flux is zero

or cases when we just want to know if the flux is positive or negative
Flux of a vector field $\vec{F}(\vec{r})$ through closed surface

- Cube: 6 flat surface piece of area $A$
- $\hat{n}$ points from inside to outside
- Vector field $\vec{F}(\vec{r})$ which is uniform on each piece, that is $\vec{F}(\vec{r}) = \vec{F}_i$ at all points on piece $i$
- Contribution of each piece is $(\vec{F}_i \cdot \hat{n}_i)A$

Flux through the cube $\Phi = \sum_{i=1}^{i=6} (\vec{F}_i \cdot \hat{n}_i)A$
What is the electric flux through the cube for $E(r) = 3.0 \times \hat{i}$?

(a) 0
(b) 12 N m$^2$/C
(c) 24 N m$^2$/C
(d) 36 N m$^2$/C
What is the electric flux through the cube for $E(r) = 3.0 \, \hat{i}$?

(a) 0  
(b) 12 N m$^2$/C  
(c) 24 N m$^2$/C  
(d) 36 N m$^2$/C
What if the vector field is not the same at all points on the flat surface piece?
What if the vector field is not the same at all points on the flat surface piece?

Rectangular flat surface in the x-y plane
Divide into tiny pieces with area $dx \, dy$
sum $\mathbf{E}(x,y) \cdot \hat{n} \, dx \, dy$ over all the pieces
two-dimensional integral
What if the surface is *not* made of flat pieces?
What if the surface is not made of flat pieces?

Divide surface into tiny pieces
If tiny enough – flat with area $dA$ and normal $n$
-- compute $(\vec{E} \cdot \hat{n})dA$
Add up the contributions from the tiny pieces

$$\int \vec{E} \cdot d\vec{A}$$
A single point charge $q$: $\vec{E}(\vec{r})$ is given by Coulomb’s law

Compute flux through spherical surface $S$ of radius $R$ centered at point charge $q$
A single point charge q: $\vec{E}(\vec{r})$ is given by Coulomb’s law

Compute flux through spherical surface S of radius R centered at point charge q

Flux integral

$$\oint \vec{E} \cdot d\vec{A}$$

$$\vec{E}(\vec{r}) \cdot \hat{n})dA$$

$$\hat{n} = \hat{r}$$

$$\left(\frac{kq\hat{r}}{R^2} \cdot \hat{n}\right)dA = \frac{kq}{R^2} dA$$

Surface integral is $kq/R^2 \ 4\pi R^2$

$$= \text{flux integral} = 4 \ \pi \ k \ q = q/\varepsilon_0$$

THE SAME FOR ANY R!

(number of flux lines is the same!)
We have shown that Coulomb’s law implies Gauss’s law for a single point charge and a spherical surface of any radius centered on the point.

What about other surfaces? Other charge distributions?
We have shown that Coulomb’s law implies Gauss’s law for a single point charge and a spherical surface of any radius centered on the point.

What about other surfaces? Other charge distributions?
For another surface $S'$, draw a spherical surface around the charge that’s completely inside $S'$.
For sphere of radius R we just did the integral to show that

\[ \oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{enc}}}{\varepsilon_0} \]

Flux line counting
Electric flux through outer surface
= electric flux through sphere

\[ \oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{enc}}}{\varepsilon_0} \]
Clicker:

What is the flux integral for a spherical surface of radius R with point charge q a distance R/2 away from the center?

(A) 0
(B) $q/(2\varepsilon_0)$
(C) $q/\varepsilon_0$
(D) $2q/\varepsilon_0$
(E) Need to do a nasty integral to evaluate this
Clicker:

What is the flux integral for a spherical surface of radius R with point charge q a distance R/2 away from the center?

(A) 0 
(B) $q/(2\varepsilon_0)$ 
(C) $q/\varepsilon_0$ 
(D) $2q/\varepsilon_0$ 
(E) Need to do a nasty integral to evaluate this
We have shown that Coulomb’s law implies Gauss’s law for a single point charge and any closed surface that contains the point charge.

What about other charge distributions? divide charge distribution inside into charge elements use principle of superposition Fluxes add and charges add – so totals are equal and Gauss’ law is verified for this case
For charge outside the surface

Flux line counting
Electric flux through surface with charge outside = zero

$q_{\text{enc}} = 0$ so Gauss law is true for this case also

$$\oint \vec{E} \cdot d\vec{A} = q_{\text{enc}}/\varepsilon_0$$
Starting from Gauss’s law, we can derive Coulomb’s law
Find field of a point charge $\mathbf{E}(r)$
Spherically symmetric charge distribution

Electric field?
flip

NO!
rotate

Same magnitude
Evaluate the flux integral on sphere of radius $R$

$$\mathbf{E}(\mathbf{r}) = E(r)\hat{r}$$

$$\oint \mathbf{E} \cdot d\mathbf{A}$$

**Gauss Law:**

$E(r) A = q/\varepsilon_0$

$E(r) = q/(4\pi R^2 \varepsilon_0) = kq/R^2$
direction is radial

= Coulomb’s law!
same argument works for any spherically symmetric charge distribution \( \vec{E}(\vec{r}) = E(r)\hat{r} \)

Use Gauss’ law to prove the shell theorems
- field outside a uniformly charged spherical shell is the same as if the charge were concentrated at a point at the center
- field inside a uniformly charged spherical shell is zero

\[ q_{enc} = Q \quad \quad q_{enc} = 0 \]
What have we gained relative to Coulomb’s law?

Determine the charge distribution from the electric field using divergence theorem

Simple way of finding the field for a given charge distribution
For a particular surface:
relates values of field on surface to the net charge inside
True for any closed surface:
relates electric field and charge distribution

\[ \oint E \cdot d\vec{A} = \frac{q_{\text{enc}}}{\varepsilon_0} \]
Gauss’ Law: another look

\[ \oint E \cdot dA = \frac{q_{\text{enc}}}{\varepsilon_0} \]

For a particular surface:
relates values of field on surface to the net charge inside
True for any closed surface:
relates electric field and charge distribution

What does it mean if the right hand side is zero?
Gauss’ Law: REVIEW

\[ \oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{enc}}}{\varepsilon_0} \]

What does it mean if the right hand side is zero?

It DOESN’T mean that the field on the surface is zero.
So in general, we can’t determine the field everywhere on the surface from just one number (the enclosed charge)

BUT, what if we knew enough about the field so that all we were missing was one number?

In particular, if the charge distribution is spherically symmetric, we already saw that we know a lot about the field just from this fact:

On the spherical surface of radius \( r \), we are only missing one number: \( E(r) \)
What have we gained by using Gauss’ law instead of Coulomb’s law?

Simple way of finding the field – works for very symmetrical charge distributions

• Wires and symmetric infinite cylinders (HRW 10 23-4)

\[ E = \frac{\lambda}{2\pi\varepsilon_0 r} \] (line of charge).

• Infinite flat uniform sheets of charge (HRW 10 23-5)

\[ E = \frac{\sigma}{2\varepsilon_0} \] (sheet of charge).
DEMO: an intriguing observation

Hollow conductor
What is the electric field inside?
DEMO: an intriguing observation

Hollow conductor

Electric field inside the cavity is ZERO
Conductors: (metal, tap water, the human body)

Some fraction of the electrons are free to move anywhere in the material

**ADDED CHARGE IS FREE TO MOVE ANYWHERE IN THE MATERIAL**
Rules for conductors in electrostatics:
• Charge density is zero everywhere in the interior of a conductor – all added charge is on the surface(s)
• the charges on the surface rearrange themselves until the electric field is zero everywhere in the interior of the conductor

Isolated spherical conductor with excess charge Q
All the charge is on the surface
How to arrange the charge to get zero field in the interior?
Electrostatics:
• Charge density is zero everywhere in the interior of a conductor – all added charge is on the surface(s)
• the charges on the surface rearrange themselves until the electric field is zero everywhere in the interior of the conductor

Isolated spherical conductor with excess charge Q
All the charge is on the surface
How to arrange the charge to get zero field in the interior?
Spread it uniformly on the surface of the sphere!
The shell theorem tells us that the field inside is zero
Electrostatics:

- Charge density is zero everywhere in the interior of a conductor – all added charge is on the surface(s).
- The charges on the surface rearrange themselves until the electric field is zero everywhere in the interior of the conductor.

Isolated spherical conductor with excess charge $Q$

All the charge is on the surface.

How to arrange the charge to get zero field in the interior?

Spread it uniformly on the surface of the sphere!

The shell theorem tells us that the field inside is zero.

Field outside: $E = \frac{kQ}{r^2}$

$\sigma = \frac{Q}{4\pi R^2}$

Magnitude of field at $r = R$ is $k \left( \frac{4\pi R^2 \sigma}{R^2} \right) = \frac{\sigma}{\varepsilon_0}$
Electrostatics:

- Charge density is zero everywhere in the interior of a conductor – all added charge is on the surface(s)
- The charges on the surface rearrange themselves until the electric field is zero everywhere in the interior of the conductor

Isolated spherical conductor with excess charge Q

All the charge is on the surface

How to arrange the charge to get zero field in the interior?

Spread it uniformly on the surface of the sphere!

The shell theorem tells us that the field inside is zero

Add a concentric spherical cavity
Electrostatics:
• Charge density is zero everywhere in the interior of a conductor – all added charge is on the surface(s)
• the charges on the surface rearrange themselves until the electric field is zero everywhere in the interior of the conductor

Isolated spherical conductor with excess charge Q
All the charge is on the surface
How to arrange the charge to get zero field in the interior?
Spread it uniformly on the surface of the sphere!
The shell theorem tells us that the field inside is zero
Add a concentric spherical cavity
Electric field is zero inside
Electrostatics:
- Charge density is zero everywhere in the interior of a conductor – all added charge is on the surface(s)
- the charges on the surface rearrange themselves until the electric field is zero everywhere in the interior of the conductor

Isolated spherical conductor with excess charge Q
All the charge is on the surface
How to arrange the charge to get zero field in the interior? Spread it uniformly on the surface of the sphere!
The shell theorem tells us that the field inside is zero
Add a concentric spherical cavity with charge q at the center
Electrostatics:
• Charge density is zero everywhere in the interior of a conductor – all added charge is on the surface(s)
• the charges on the surface rearrange themselves until the electric field is zero everywhere in the interior of the conductor

Isolated spherical conductor with excess charge Q
All the charge is on the surface
How to arrange the charge to get zero field in the interior? Spread it uniformly on the surface of the sphere!
The shell theorem tells us that the field inside is zero
Add a concentric spherical cavity with charge q at the center
Q redistributes so that –q is spread uniformly on the inner surface and Q+q is spread uniformly on the outer surface
Electrostatics:
• Charge density is zero everywhere in the interior of a conductor – all added charge is on the surface(s)
• the charges on the surface rearrange themselves until the electric field is zero everywhere in the interior of the conductor

Isolated conductor of any shape with excess charge Q
All the charge is on the surface
How to arrange the charge to get zero field in the interior?
Electrostatics:
- Charge density is zero everywhere in the interior of a conductor – all added charge is on the surface(s)
- the charges on the surface rearrange themselves until the electric field is zero everywhere in the interior of the conductor

Isolated conductor of any shape with excess charge Q
All the charge is on the surface
How to arrange the charge to get zero field in the interior?
Depends on the shape
Electrostatics:
• Charge density is zero everywhere in the interior of a conductor – all added charge is on the surface(s)
• the charges on the surface rearrange themselves until the electric field is zero everywhere in the interior of the conductor

Isolated conductor of any shape with excess charge Q
All the charge is on the surface
How to arrange the charge to get zero field in the interior?
Depends on the shape
Add a cavity of any shape
Electrostatics:
• Charge density is zero everywhere in the interior of a conductor – all added charge is on the surface(s)
• the charges on the surface rearrange themselves until the electric field is zero everywhere in the interior of the conductor

Isolated conductor of any shape with excess charge Q
All the charge is on the surface
How to arrange the charge to get zero field in the interior?
Depends on the shape
Add a cavity of any shape
Electric field is zero inside
Electrostatics:

- Charge density is zero everywhere in the interior of a conductor – all added charge is on the surface(s)
- the charges on the surface rearrange themselves until the electric field is zero everywhere in the interior of the conductor

Isolated conductor of any shape with excess charge \( Q \)
All the charge is on the surface
How to arrange the charge to get zero field in the interior?
Depends on the shape
Add a cavity of any shape with charge \( q \) at the center
All points on dashed surface are in the interior of the conductor $E=0$ so electric flux through dashed surface is zero.

By Gauss’ law, net charge inside dashed surface is zero.

So there must be a total of $-q$ on the surface of the crescent moon shaped cavity.
Electrostatics:
• Charge density is zero everywhere in the interior of a conductor – all added charge is on the surface(s)
• the charges on the surface rearrange themselves until the electric field is zero everywhere in the interior of the conductor

Isolated conductor of any shape with excess charge Q
All the charge is on the surface
How to arrange the charge to get zero field in the interior?
Depends on the shape
Add a cavity of any shape with charge q at the center
Q redistributes so that −q is spread on the inner surface and Q+q is spread on the outer surface