Inertia tensor for continuous system

\[ \text{rod lying in } x-y \text{ plane} \]
\[ \text{length } = 2\text{L} \]

\[ \begin{align*}
I_{xx} &= \int dm \left( y^2 + z^2 \right) = \int dm \cdot y^2 \\
&= \int_0^L dm \ y^2 = \frac{1}{2} \int_0^{2\text{L}} dy \ y^2 \\
&= \left[ \frac{1}{3} \text{L}^3 \right]_0^L = \frac{\text{L}^3}{3} \\

I_{yy} &= \frac{\text{L}^3}{3} \\
I_{zz} &= 2 \int dm \ y^2 = 2 \left( \frac{\text{L}^3}{3} \right) \\
&= \frac{2\text{L}^3}{3}
\end{align*} \]

Similarly, \[ I_{yy} = \frac{\text{L}^3}{3} \]

Notice, \[ I_{xx} + I_{yy} = I_{zz} \]

This is a result known as perpendicular axis theorem

\[ I_{xz} = I_{zx} = I_{yz} = I_{zy} = 0 \]

\[ I_{xy} = I_{yx} = -\int xy \ dm = -\int dm y^2 = -\frac{\text{L}^3}{3} \]

\[ \text{Notice: if I consider a symmetric system, all diagonal terms would be 0 by symmetry.} \]
For every element with a positive contribution, there is an element with equal negative contribution. The net contribution of these 2 elements is 0.