5.7

\[ T = 2N \cos \theta + Mg \]
\[ mg \cos \theta - N = \frac{mv^2}{R} \]

By energy conservation:
\[ \frac{1}{2}mv^2 = mg \cos \theta (1 - \cos \theta) \]

Find \( \cos \theta_{\text{max}} \) for \( T = 0 \)

5.10

Weight is because of 2 things:
1. Chain already on the scale = \( 2g \cdot x \)
2. Part of chain hitting the scale = \( \frac{dv}{dt} = \frac{v \cdot dm}{dt} = \frac{v \cdot 2dx}{dt} = 2v^2 \)

From kinematics, \( v = \sqrt{2g \cdot x} \)

5.13

(a) \[ U = -\frac{GM_1M_2}{r_{12}} \]

(b) Change in potential energy = \(-\) (change in kinetic energy)
(a) \[ F = \frac{dP}{dt} = \frac{vdm}{dt} \]

\[ P = VF = \frac{v^2dm}{dt} \]

(b) Rate of increase of K.E

\[ = \frac{dK}{dt} = \frac{1}{2} \frac{d}{dt} v^2 = \frac{1}{2} P \]

---

(5.19)

(a) \[ \frac{dP}{dt} = \frac{v_0dm}{dt} = 2v_0^2 \]

\[ \Rightarrow F = 2yv_0 + 2v_0^2 \]

(b) \[ P = Fv_0 \]

---

(6.1)

\[ F_x = -\frac{2GmM}{r^2} \cos \theta \]

\[ r^2 = x^2 + a^2 \]

\[ \Rightarrow F = -\frac{2GmM}{x^2+a^2} \times \frac{x}{\sqrt{x^2+a^2}} \]

for small x, ignoring \( x^2 \) terms

\[ F = \left( \frac{2GmM}{a^3} \right) x \approx F = -kx \]
We can see by symmetry that
\[ x_1 = \pm x_4 \quad \text{and} \quad x_2 = \pm x_3 \]

Since c.o.m is at rest, we have
\[ X = \sum_{i=1}^{4} m_i x_i = 0 \]

For mass \( m \),
\[ m \ddot{x}_i = k (x_i - x_j) \]
\[ \ddot{x}_i = \omega^2 x_i \quad \Rightarrow \quad \omega^2 x_i = \frac{k}{m} (x_i - x_j) = \omega_0^2 (x_i - x_j) \]
\[ \Rightarrow \ \beta x_i = (x_i - x_j) \]

Similarly for the other masses:
\[ \beta x_2 = (2x_2 - x_1, -x_4) \quad ; \quad \beta x_3 = (2x_3 - x_2, -x_4) \quad ; \quad \beta x_4 = (x_4 - x_3) \]

Solving these for the constraint identified above gives the required resonant frequency.

In general, one **CANNOT** replace any springs by a rod unless\[ |x_1| = |x_2| \]
(a) When walls are stationary,

\[ \Delta P = 2m \Delta v \]

\[ \Rightarrow F = \frac{\Delta P}{\Delta t} = \frac{2m \Delta v}{\Delta t} = \frac{mv^2}{l} \]

(b) When one wall is moving,

Change in speed of the ball = 2v per collision with the moving wall.

Now, \( \Delta T = \frac{2T}{v} \)

\[ \Rightarrow \frac{dv}{dt} = \frac{2V}{2x/v} = \frac{uv}{x} \]

\[ \frac{dv}{dx} = \frac{dv/dt}{dx/dt} = \frac{1}{V} \frac{dv}{dt} \quad \text{(here } x \text{ is decreasing)} \]

(c) \[ F = \frac{dP}{dt} \]

For 6.7 and 6.8,

Use energy and momentum conservation.