HARMONICS

The objectives of this experiment are:

- To understand how individual waves add to produce another wave form

- To see how frequency ratios determine the musical scale system.

APPARATUS: PC, Fourier synthesis Java program.

INTRODUCTION

In this experiment we will study how sound waves can add together to produce different wave forms that we sense as "timbre". In particular, there is a class of waves that are important to the study of sound and especially music. These are called "harmonics".

In order to understand the concept of harmonics, let us first consider a simple sinusoidal wave having the form of

$$y = A\sin(\omega t).$$

This tells us that the wave has an amplitude equal to A and it is periodic. That is, the wave of frequency, f, repeating itself with a period, T.

$$\omega = 2\pi f = \frac{2\pi}{T}$$

Sound waves often can be modeled by a simple sine function that describes the sound pressure as a function of time. But, even more common, we find that sound emanating from musical instruments and many other natural sources have a basic or *fundamental* frequency that is accompanied by other frequencies that are integer multiples of the fundamental frequency. These higher frequencies are called *overtones*; and when they are integral multiples of the fundamental, they are called harmonics. The fundamental is called the first harmonic. The next is called the second harmonic or first overtone.

A sound wave comprised of a fundamental frequency accompanied by other harmonics of various amplitudes will have a certain tone quality or timbre or *tone color*. Varying the relative amplitudes of the harmonics will change the timbre. This is why a "middle C" note played on a piano will sound different than the "middle C" note from a violin. If we analyze the spectrum of the sound wave, we can see the difference in the relative harmonics. Note that a high quality tuning fork will produce only a fundamental frequency with virtually no overtones.

In other experiments you have seen how harmonics are produced whether by standing waves on a vibrating string or in a resonating tube. The physical implication is that the harmonics must be in phase. Special music synthesizers can alter the phase of harmonics

which will produce different musical qualities. We can express a waveform F(t) simply as the series addition of harmonics: $F(t) = \sum_{n=1}^{\infty} A_n \sin(2\pi f_n t)$

Few people are capable of recognizing the true pitch of a single musical tone, but many are able to tell what the ratio of the frequencies of two tones is. Most people can recognize that certain tones when sounded together produce a pleasing effect while other combinations give rise to an unpleasant sensation. Through experience certain frequencies have been selected to construct a musical system of pitch intervals and scales. By the term, *musical interval*, we mean the ratio between the frequencies of any two tones.

For instance one musical sound may have a frequency of 330 Hz and another at 660 Hz. They are in a ratio of 2 to 1 which produces a very pleasant sound that is called an *octave*. (The name originates from musical scale that divides this interval into eight tones. Octal means eight) There is a family of such intervals for which the ratios of frequencies have been calculated. These musical intervals include:

Unison	1:1	Major Third	5:4
Octave	2:1	Minor Third	6:5
Fifth	3:2	Major Sixth	8:5
Fourth	4:3	Minor Sixth	5:3

As we said there are pairs of notes that produce agreeable sounds. There are also groups of three notes that produce similar effects. Such triads have a simple relation with the *major chord* being the most important. These notes are in a ratio of 4:5:6. Below is the musical scale showing the placement of the major chord on the *major diatonic scale*.

```
   C       D       E       F       G       A       B       C'      D'

 4....  .........  ....5....  .........  ....6

              4....  .........  ....5....  .........  ....6

                           4....  .........  ....5....  .........  ....6
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Here is the ratio of the frequencies of the major diatonic scale. f is the frequency of the initial note, in this case C, which is called the *tonic*.

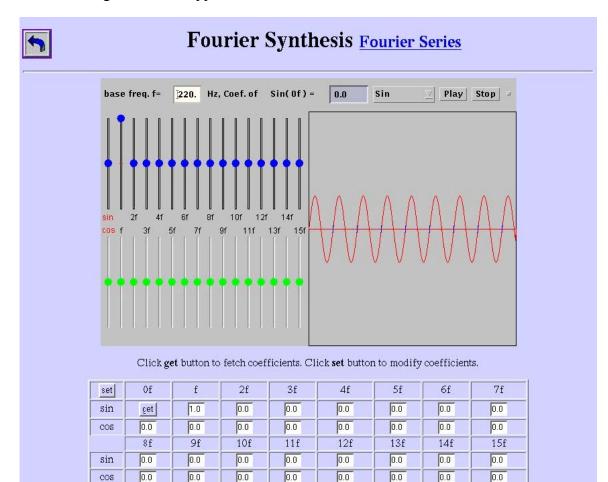| Note:	C	D	E	F	G	A	B	C'
Frequency	f	$(9/8)f$	$(5/4)f$	$(4/3)f$	$(3/2)f$	$(5/3)f$	$(15/8)f$	$2f$

In addition to the major triad and chord there is a *minor* triad and chord which bear the ratio of 10:12:15. The ratios for that system are

Note:	C	D	E	F	G	A	B	C'
Frequency	f	$(9/8)f$	$(6/5)f$	$(4/3)f$	$(3/2)f$	$(8/5)f$	$(9/5)f$	$2f$

"FOURIER SYNTHESIS" Web Page

You will use a Web Java program for this lab. Point your web browser to http://labspc200.rutgers.edu/mirror/Fourier-synthesis/fourier.htm

The following screen will appear.

Study the above illustration and read below the explanation of the various control buttons. Start at the top, left of the figure.

Base (Fundamental) Freq. Selector:
Enter the value of the fundamental (base) frequency in the text box. Adjust this to a comfortable level by selecting the Sine button and play it for different values of the base frequency. Be sure to type "Enter" with the cursor in the text box when changing the base frequency and note that the waveform plotted corresponds to the new freq.

Waveform Pull Down Menu:
Use the pull down menu to select one of several preset wave forms. Choices are sine, cosine, sawtooth, square, rectifier, and user. When one of these is selected the amplitude of each harmonic is set to the correct amplitude. Note that an infinite number of harmonics is needed to produce perfect square and sawtooth wave forms.

Amplitude Sliders:
 These are the horizontal sliders at the side of the waveform graph window. They are used to adjust the amplitude of each harmonic. Note that the amplitude value ranges from 0 to 100% as you move the slider.

Playing the Sound Wave:
To listen to the sound generated by your waveform select the **Play** button at the top with the mouse cursor (click once).

Entering Numerical Values for Harmonic Amplitudes:
 You can enter numerical values for the harmonic amplitudes using the table below the main plot. Select "User" under the waveform pull down menu. Enter a value in the box appropriate to the harmonic number you want to change and click on the "set" button.

PROCEDURE:

A. After you have followed the above instructions to initialize the program, add a few harmonics to the fundamental frequency of a sine wave. Play it and listen to how the timbre changes as higher harmonics are added.

B. Select the Sawtooth Wave. If we had an infinite number of harmonics, a perfect Sawtooth wave form would be reproduced. You would see a perfectly sharp edge followed by a perfectly straight slanted line leading to the next sawtooth edge. What is the form of the mathematical series for the waveform? That is,

$$F(t) = \sum_{n=1}^{\infty} A_n \sin(2\pi f_n t)$$

What does A_n look like?

Play the sawtooth waveform. Then, reset the highest harmonic to zero. Play it again and take note of the change in timbre. Then, reset the next highest harmonic to zero, and listen to the resulting sound. Continue doing this until you get to just the fundamental.

C. Construct a special waveform that has the harmonics of the form:

$$F = \sum_{\substack{n=1,3,\dots \\ (odd)}}^{13} \frac{1}{n} \sin(2\pi f_n t)$$

where the even harmonics are zero. Watch how the waveform changes as you add each odd harmonic. Try listening to each waveform. If you had a truly infinite series, what would the waveform be?

D. Next repeat the previous exercise but apply harmonic amplitudes that fall of as the square of n:

$$F = \sum_{\substack{n=1,3,\dots \\ (odd)}}^{13} \frac{1}{n^2} \sin(2\pi f_n t)$$

Watch how the waveform changes as you add each odd harmonic. Try listening to each waveform. If you had a truly infinite series, what would the waveform be?

E. Now, zero all of the harmonic amplitudes and set the f_9 and f_{10} to a full amplitudes. Play it and listen carefully. The wavering sound you hear is called "beats". Look at the screen. What is causing the wavering sound? To some ears this will sound like a raspy, bad speaker.

F. Finally, examine the wave forms of the musical intervals. Sketch the wave form and take note of the beat pattern. Which have the most pleasant sounds? The least? Is this reflected in the wave form?

NOTES

EXPERIMENT 6

HARMONICS

Name: _____ Section: _____

Partner: _____ Date: _____

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MUSICAL INTERVAL   WAVE FORM

OCTAVE:

FIFTH

FOURTH

MAJOR SIXTH

MAJOR THIRD

MINOR SIXTH

MINOR THIRD