MECHANICAL RESONANCE

The objectives of this experiment are:

- To study the resonance behavior of a mechanical oscillator.

- To measure the oscillator amplitude and phase as a function of frequency.

OSCILLATIONS

T he loudness or intensity of sound emitted by a mechanical body depends upon the amplitude of the vibrating motion. Most sources of sound such as vocal cords, a guitar string, or the lips of a horn player, would produce, by themselves, a very faint sound. There is, however, a simple physical system or means that greatly amplifies this small amount of acoustical energy. This process is known as acoustical resonance.

In order to understand the importance of this phenomenon, we have to study the nature of mechanical resonance. Any body that is free to vibrate has natural periods of oscillation. A thin rod of steel, if it is struck, will oscillate back and forth if it is supported at one end. The oscillation will, of course, depend on its mass, length, and other mechanical properties of the rod. A string or wire that is held taught at its ends will vibrate when plucked. Its natural frequencies depend on the density, diameter, length, and tension of the wire. In both cases they oscillate for a time at their own peculiar, natural periods. They stop vibrating because they are emitting, and thus losing, energy in the form of acoustical waves.

If the steel rod were subjected to a vibrating source of energy that has the same frequency as the natural frequency of the rod, it will start to oscillate in sympathy with the external source. If conditions are in a certain way, the rod's oscillations can be quite large. If a person sings the correct pitch that corresponds to the frequency of the wire, the wire will start to vibrate and emit a similar pitch. The purpose of this experiment is to investigate and determine those special conditions that will produce the phenomenon of *resonance*. In particular we will study the case in which the vibrating object loses energy fairly quickly - a *damped* oscillator.

One type of damped mechanical oscillator is shown in Fig. 1a. Here, the upper end of a spring (where spring constant is k) is held fixed, and a mass (m) is attached to its lower end. When the mass is at rest, its weight is balanced by an upward force from the stretched spring. When the mass is displaced from the equilibrium position and released, it oscillates. The mass is immersed in a fluid, so that a resistive force also acts on it. This force opposes the motion and is proportional to the object's speed. Therefore, the oscillations are damped and die out.
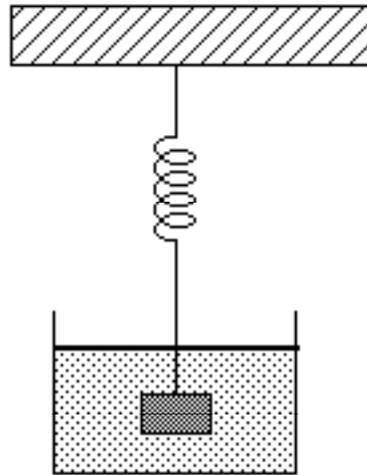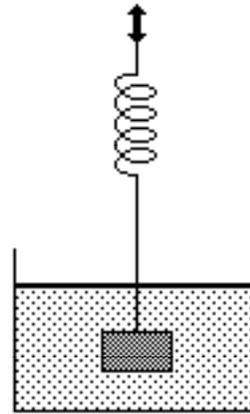
FIGURE 1A   FIGURE 1B

FORCED OSCILLATIONS

Suppose now that the spring's upper end is not fixed, but is moved up and down in simple harmonic motion, as indicated in Fig. 1b. The spring, and therefore also the suspended mass, is now subject to an additional applied force that varies sinusoidally. We now have a damped oscillator driven by an external oscillator. The driving frequency $\underline{f}$ of the external oscillator need not be the same as that of the damped oscillator alone, but if the damped oscillator is driven near its natural oscillation frequency f_O ($f = f_O$), then we have "resonance", and the mass will oscillate with large displacement (and velocity) amplitude. In this experiment we observe displacement of the oscillating mass, not its velocity.

The resonance behavior is manifested in two different ways. The amplitude resonance is frequently more dramatic, but the phase resonance, which is the angular shift between the external driver and the oscillating object, may sometimes be more clear.

- The oscillation $\underline{displacement,}$ **amplitude**, is at a maximum near $f = f_o = \dfrac{1}{2\pi}\sqrt{k/m}$ and smaller off resonance, both for $f < f_O$ and for $f > f_O$. In fact, the damping shifts the resonant frequency down to $f_o = \dfrac{1}{2\pi}\sqrt{k/m - R^2/2m^2}$. On the other hand, the $\underline{velocity}$ amplitude (which we do not observe directly in this experiment) is maximum at f_o, too. Neither displacement nor velocity curves are perfectly symmetric above and below their peaks.

- The $\underline{phase}$ of $\underline{displacement}$ oscillation relative to the driving force shifts by 180° as the driving frequency varies through resonance.

It is easy to see why there must be such a phase shift. First, suppose that $f \ll f_o$. The top end of the spring is then oscillating so slowly that the mass at the lower end simply follows this motion. The phase difference between the external driving force and the mass's displacement is zero for $f \ll f_o$. Now consider the other extreme, with $f \gg f_o$. The mass cannot keep pace with the rapidly varying force at the spring's top end. When the top end is at its highest point, the lower end is at its lowest point, and vice versa: the driving force and displacement are 180° out of phase. Figure 2B shows how the displacement phase shift θ goes from 0° to 180° as the frequency increases.

GENERAL DESCRIPTION OF FREQUENCY DEPENDENCE

The oscillating mass is subject to three forces:

- The applied force sinusoidal force F_a of the driving oscillator

- A linear restoring force, -kx, from the compressed of stretched spring. (The displacement x is measured from the equilibrium position at which the weight is balanced by the force of the spring.)

- A resistive force, -Rv, which is opposite to the direction of the mass's velocity and proportional to its magnitude.

A detailed mathematical derivation of the behavior is given in most textbooks. Here we give only the principal results in order to give you an intuitive feeling for this important phenomenon.

After start-up when the system settles down, steady state is achieved and all changes of displacement and velocity are at the driving frequency f. The natural (undriven, undamped) oscillation frequency of the system, f_o, controls the amplitude and phase of the system.

The oscillating mass has greatest displacement and speed when it is driven at its natural angular oscillation frequency ω_o = sqrt(k/m) (radians/second).   The displacement resonance shown in Fig. 2 below exhibits: (a) a sharp peak in the amplitude of the displacement and (b) a phase shift (between y and F_a) that is 0° at low frequencies, 180° at high frequencies, and 90° near the resonance frequency.

amplitude vs. frequency

little damping

much damping
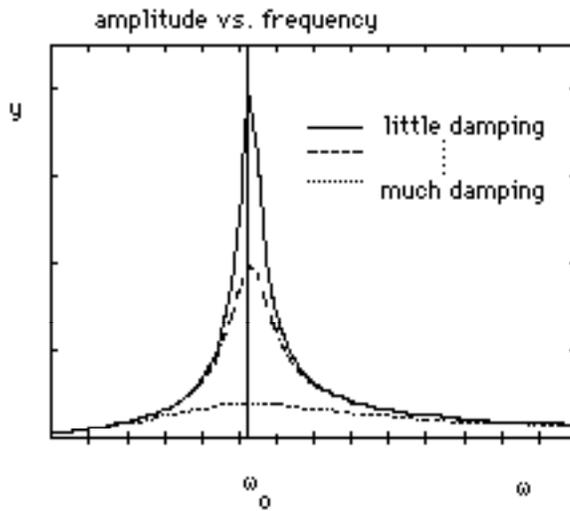
phase vs. frequency
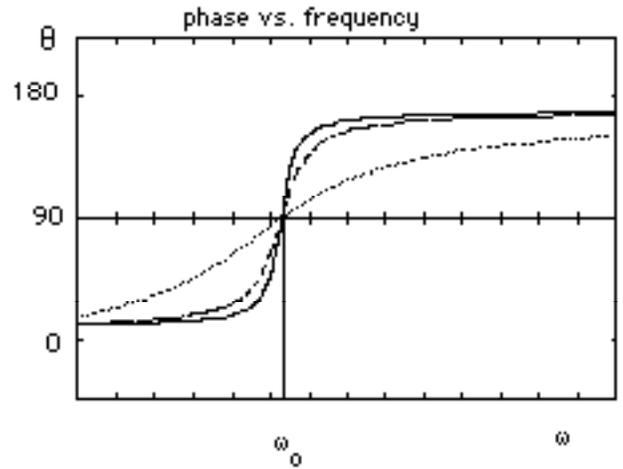
FIGURE 2A   **FIGURE 2B**

In this experiment, we measure directly two quantities shown in Fig. 2a/b as a function of frequency:

   • The **amplitude of the <u>displacement</u>** (not the velocity).

   • The **phase angle θ between the applied force and the <u>displacement</u>** .

The expression for the displacement amplitude and phase θ is

$$x = \frac{F_m/\omega}{\sqrt{\left(\omega m - \dfrac{k}{\omega}\right)^2 + R^2}} \sin(\omega t - \theta) \ (10)$$

$$\tan \theta = \frac{R}{\left(\dfrac{k}{\omega} - \omega m\right)}$$

Were it not for the 1/ω factor, the maximum of the displacement amplitude would occur
     at
ω0= sqrt(k/m). In fact, as noted above, it is shifted down a little in angular frequency to
sqrt(k/m - R²/2m²). In general, for small damping, the amplitude resonance curve looks
very similar near the resonance to Fig. 2a for the resonance.

PROCEDURE

1.   <u>Calculate</u> (predict) the **natural frequency**, $f_\circ = \dfrac{\omega_\circ}{2\pi} = \dfrac{1}{2\pi}\sqrt{\dfrac{k}{m}}$.   Do this by first

   measuring the length the spring stretches with the mass attached. Support the mass

with your hand and measure the length of the spring. Lower and steady the mass; measure the length of the spring when stretched. The difference between these two measurements is x. Use Hooke's Law to get *k:* F= mg =-*k*x. Knowing x, you can calculate the natural frequency as you did in the preliminary question. <u>You don't need to measure m</u>!

2. <u>Measure</u> the **natural frequency**, $f_{o,exp}$ by pulling down (don't overstretch) and releasing the bob. You must do this by lifting the bob out and holding it away from the damping magnet

3. Calculate the ratio between your experimental result for f_o and the theoretical value determined above.

4. Level and zero the apparatus so that the bob does not rub against anything. Your instructor will describe this procedure.

5. Measure the displacement amplitude, x_m, and displacement phase angle θ (between the driving force and the displacement) as a function of the driving frequency. Start with the lowest frequency. The angle will be small, close to 10° or 20°. AS you increase the driving frequency, you will go past 90° approaching 180°. Graph the data as you go along to save time.

    Measure the frequency of oscillation by timing several oscillations. The period, T, of oscillation is the total time for the oscillations divided by the number of oscillations. Remember, f = 1/T. The driving frequency is controlled by rotating a knob on the apparatus. Measure the displacement with the ruler provided on the apparatus. The displacement is one-half of the total top-to-bottom distance. The phase angle is read directly by looking down through the protractor mounted on the red plastic plate. The revolving LED (<u>L</u>ight <u>E</u>mitting <u>D</u>iode) will light at the appropriate spot marking the phase between the driving force and the displacement.

    Note: you must wait for the system to reach its steady state before taking readings. If you change the driving frequency rapidly, you will notice that the LED will not mark a consistent phase angle. Wait until the phase LED is steady before recording data.

6. Plot x_m and θ each as a function of frequency, displaced vertically, in the fashion of Fig. 2a and 2b. Find and mark on the graphs the frequency corresponding to the peak in the resonance curve and to phase $\theta = 90°$. Then, compare these frequencies to your experimental natural frequency result in part 2. Use CricketGraph software if computers are available.

NOTES

MECHANICAL RESONANCE

Name: _____

Partner: _____ Date:_____

Free Oscillations   Show calculations.

Theoretical natural frequency calculation from stretch (See Preliminary Question)

x_o _____ $f_{o,th}$ _____

Experimentally derived derived frequency:

Δt _____   # oscillations _____ $T_{o,exp}$ _____ $f_{o,exp}$ _____

Compare theoretical and experimental values: $f_{o,exp} / f_{o,th}$ = _____

Forced oscillations   Frequency variation of displacement amplitude and phase

Collect enough data to make two graphs: one of amplitude as a function of frequency; the second, displacement phase as a function of frequency.

# OF CYCLES N	TIME (sec)	ONE PERIOD T (sec)	FREQUENCY f (Hz)	AMPLITUDE x_m	PHASE θ

From the two graphs, determine resonance frequencies and compare them with your experimentally measured frequency, $f_{o,exp}$.

WARNING: you must apply the proper standards for graphs.

Displacement amplitude:

Resonance frequency (from experimental frequency plot). $f_{o,amp}$ _____

Ratio $f_{o,amp}$ / $f_{o,exp}$: _____

Displacement phase:

Resonance frequency (from experimental frequency plot). $f_{o,\theta}$ _____

Ratio $f_{o,\theta}$ / $f_{o,exp}$: _____

List sources of error for the two graphed data sets:

-
-
-

QUESTION ONE

Is the plot of $x_m(f)$ asymmetrical in the sense consistent with equation 10?

Explain:

Attach your graphs to this report form.