SIMPLE HARMONIC MOTION

Name:____ Section:____________

Partner:____ Date:____________

Spring Constant, k: Measure and record the change in the spring's length for each change in the added weight. Using MKS units, enter the data in the table. Use `Excel` to plot F versus Δx. Then fit a straight line to your data. How will this fit tell you the value of k?

TABLE 1. Hooke's Law: $F = -k\Delta x$

x (m)	Δx (m)	Δm (kg)	$F = \Delta mg$ (N)
	0	0	0

From a simple fit, $k =$________________________ (include units!)

SHM Period: Record the total mass and measured period below:

Total Mass =__________________ Period, $T_{\text{meas}} =$____________________

Now calculate the theoretical period from your measurement of k and the mass. Compare the theoretical and measured values.

Period, $T_{\text{theor}} =$________________ Ratio, $T_{\text{meas}}/T_{\text{theor}} =$__________________

Equation 6 assumes the spring is massless. In fact, we should add 1/3 the mass of the spring to m. Take the spring mass to be 30 g. Does this improve the agreement?

Period, $T_{\text{theor}} =$________________ Ratio, $T_{\text{meas}}/T_{\text{theor}} =$__________________

Displacement/Velocity Magnitude Relationship: Pick any convenient time on the displacement and velocity graphs, record the values of t, x and v plus the value of A. Substitute your measured values of k, A and x into Eqn. (3) to obtain a predicted velocity. Compare this with the measured velocity

$t =$____________, $x =$____________, $v_{\text{meas}} =$____________, $A =$____________.

$v_{\text{pred}} =$____________ (show work here)

$\frac{v_{\text{meas}}}{v_{\text{pred}}} =$____________

Displacement/Velocity/Acceleration Phase Relationships: From your pairs of graphs for displacement-velocity and displacement-acceleration, it is obvious that they are all sinusoidal curves. We want to know their phase relationship or relative phase shift. Example: if the maximum peaks coincide, they are in phase; the phase shift, $\Delta theta$ is 0 radians or 0°. If the maximum peaks correspond to the minimum of another they are shifted by $\Delta theta = \pm\pi radians = \pm 180°$.

Record the time of the peaks for distance (displacement) and velocity. Select the first displacement peak, record its time. Then find the closest velocity peak and record its time. Repeat this for the next pair of peaks. Use the period to calculate the phase relationship, $\Delta\theta$ in degrees.

TABLE 2. Displacement-Velocity Phase Analysis

	Peak pair #1	Peak pair #2	Peak pair #3
Time of Displacement Max.			
Time of Velocity Max.			
$\Delta\theta = 360°(T_V - T_D)/T$			

Determine the average $\Delta\theta =$______________________

Does the velocity lead or lag the displacement, i.e., does it reach its peak value before or after the (nearest) displacement peak?

Do the same analysis for displacement and acceleration.

TABLE 3. Displacement-Acceleration Phase Analysis

	Peak pair #1	Peak pair #2	Peak pair #3
Time of Displacement Max.			
Time of Acceleration Max.			
$\Delta\theta = 360°(T_V - T_D)/T$			

Determine the average $\Delta\theta =$ ____________________

Does the acceleration lead or lag the displacement?

From your analysis complete the following table. For each case, answer whether the displacement, X, is a negative maximum, at 0, or a positive maximum.

TABLE 4. Summary of Phase Analysis

Case	Displacement (Neg. maximum, 0, or Pos. maximum?)
Velocity is positive Maximum	X is . . .
Velocity is 0	X is . . .
Velocity is negative maximum	X is . . .
Acceleration is positive Maximum	X is . . .
Acceleration is 0	X is . . .
Acceleration is negative maximum	X is . . .

Mass and Amplitude Effects on the Period: Go back and measure the period for several values of the mass. Record your data. Compare your measurements with the theoretical value from Eqn. (5).

TABLE 5. Mass Dependence

Measured Period T_{meas}	Mass m	Predicted Period T_{pred}	Ratio $T_{\text{meas}}/T_{\text{pred}}$

Measure the period for several amplitudes without changing the mass. Is there any change in the period? What is the theoretical period for your setup?

TABLE 6. Amplitude Dependence

Amplitude	Period

`Interactive Physics` Simulation of The Simple Pendulum

Dependence of Period on Mass, Gravity, and Length.

Use the simulation to verify Eqn. (12). To vary the mass click on the circle, choose Properties from the Window menu and type a new value for the mass in the appropriate box. To vary the value of g, choose Gravity from the World menu and type in a new value for g. To vary the length of the pendulum, click on the rod and type in a new value in the properties box. Measure the period by recording the difference in times between when $|v| = 0$. (Be careful, it's easy to make a mistake here!)

TABLE 7. Verify Period Dependence on m, g, and L

Mass (kg)	g (m/s^2)	Length (m)	Measured Period (s)	Predicted Period (s) (eqn. 12)
50	9.8	3		
0.5	39.2	3		
0.5	2.45	3		
0.5	9.8	12		
0.5	9.8	0.75		
0.5	9.8	3		

Dependence of Period on Amplitude (Starting Angle)

Eqn. (12) was derived assuming the amplitude of oscillation was small so that we can make the approximation $\sin\theta \approx \theta$. Use your simulation to see under what conditions this approximation is true. To change the starting angle, calculate the appropriate x or y value and enter it under properties. (The simulation does not calculate θ, so you will have to calculate the initial x or y coordinates, i.e., x_0 and y_0, of the circle). In the last column of the table compute the ratio of the period value you found from the simulation to the period of a simple pendulum (small angle approximation). Write that value here:

Period, T_{simple} =____________________________

TABLE 8. Period Dependence on Amplitude

x_0	y_0	$\theta_0\,(^\circ)$	$\theta/\sin\theta$	Period (s)	Ratio of periods
		10			
		20			
		45			
		60			
		90			
		135			

What is the largest starting angle for which the measured period will agree with the small angle period within 1% (find where $\sin\theta$ differs from θ by 1%)? ____________

Acceleration

Predict the direction of the acceleration of the mass by drawing an arrow on the each of the circles below which represent the starting point of the pendulum, a point half way down, and the bottom of the swing.

Now check your predictions by choosing the acceleration vector under the Define menu. This will place the acceleration vector on the circle during the simulation. Explain why the acceleration points in the direction it does for each of the three cases

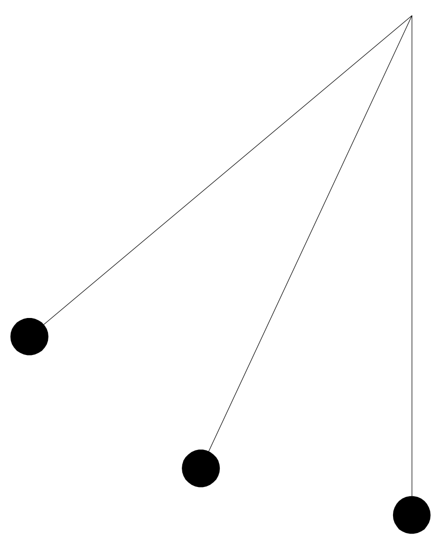

FIGURE 1. Simple Pendulum