Basic Math - Review
Due Sept 12 2006.

1 Vectors

1. Find the angle between two vectors given in spherical coordinates by Euler angles \( \theta(\theta') \) and \( \phi(\phi') \). Here, as usual, \( \theta \) represents the altitude and \( \phi \) the longitude.

2. Prove that \( b_\alpha \) (three component quantity) is vector if \( a_\beta \) (three component quantity) is vector and \( a \cdot b = \text{const} \) (is invariant under rotations).

3. What is the determinant of the rotation transformation? Prove your statement.

4. Find the average value of the following expressions over the directions of the unit vector \( n \).
   a. \( (a \cdot n)^2 \)
   b. \( (a \cdot n)(b \cdot n) \)
   c. \( (a \cdot n)n \)
   d. \( (a \times n)^2 \)
   e. \( (a \cdot n)(b \cdot n)(c \cdot n)(d \cdot n) \)

2 Calculus

1. Write down the expressions for \( \nabla \phi \), \( \nabla \cdot A \) and \( \nabla \times A \) in (a) spherical and (b) cylindrical coordinate systems.

2. Compute \( \nabla \cdot r \), \( \nabla \times r \) and \( \nabla(r \cdot p) \) where \( r \) is radius vector and \( p \) is constant.

3. Compute \( \nabla \times (\omega \times r) \) where \( \omega \) is constant vector.

4. Compute \( \nabla \phi(r) \), \( \nabla(\phi r) \), \( \nabla \times (\phi r) \) where \( \phi(r) \) depends only on the length of radius vector \( r \).

5. Transform integral over the volume \( \int (\nabla \phi \cdot \nabla \times A) dV \) into the surface integral.

6. Compute vector integrals \( \oint (a \cdot n)r dS \) and \( \oint (a \cdot r)n dS \) where \( a \) is constant and \( n \) is normal to the surface and \( r \) is radius vector.