Midterm Exam, Quantum Mechanics 501, Rutgers

October 28, 2015

1. **Short questions:**
   (a) What is a pure state? How does its density matrix look like?
   (b) When you make measurement on a pure state, are you assured of getting precise values for observables?
   (c) What is the form of the density matrix for mixed (non-pure) state?
   (d) If $|i\rangle$ and $|j\rangle$ are eigenkets of Hermitian operator A. Under what conditions is $|i\rangle + |j\rangle$ an eigenket of A?
   (e) Without explicit calculation, sketch the wavefunction of the lowest two eigenstates of a particle in a potential

$$V(x) = \begin{cases} kx & x \geq 0 \\ \infty & x < 0 \end{cases} \tag{1}$$

   taking care to show how the shape and amplitude vary with position.

2. Two quantum operators have the matrix representation

$$A = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad B = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \tag{2}$$

   (a) A system is in quantum state $|\psi\rangle$ that is in an eigenfunction of operator A, corresponding to eigenvalue -1. Then for this state, what are $\langle A\rangle$ and ΔA?
   (b) First B is measured and the result is $b = -1$. What is the state of the system after the measurement?
   (c) Immediately afterwards, A is measured. What is the probability to find $a = 1$?
   (d) Assuming that $a = 1$ was indeed found in (c), what is the state of the system after the measurement of A?

3. The wavefunction of a particle of mass m is in a 1D potential $V(x)$ is

$$\psi(x) = \begin{cases} Axe^{-ax} & x \geq 0 \\ 0 & x < 0 \end{cases} \tag{3}$$

   (a) Assuming the particle is in an eigenstate of the Hamiltonian, find the potential $V(x)$ and the total energy E for this state.

   (b) Find the potential energy expectation value $\langle V \rangle$ for this state

   (c) Find the expectation value of the kinetic energy for this state.

4. The eigenstates, which are accesible to a single electron, have energies ε_0, ε_1 and ε_2 and their states are $|0\rangle$, $|1\rangle$ and $|2\rangle$. When two electrons are introduced in such system, what are possible wave-functions of the system of two electrons, if we neglect interaction between the two electrons?

   (a) How many possible states can you write down, which have correct statistics? Write them down.

   (b) What are the energies of these states?

   (c) Is the state $|0\rangle \otimes |1\rangle$ a valid wave function of such system? Why (not)?