Midterm Exam, Quantum Mechanics 501, Rutgers

October 29, 2014

1) An electron ($m = 0.511\times 10^6\,\mathrm{eV/c^2}$) is bound in a parabolically shaped one-dimensional potential. The parameters of the potential can be found from the fact that the electron feels potential of 1eV when it is 5 Å away from the center.

   – What minimum total energy (in eV) can it have?
   – What wavelength of light (in nm) will be strongly absorbed by this electron?

   Useful constants: $\hbar c \approx 197\mathrm{eV\,nm}$ and mass of the electron $m = 0.511 \times 10^6\,\mathrm{eV/c^2}$

2) Now suppose the electron is in superposition of the first two excited states $|\psi(t=0)\rangle = \frac{1}{\sqrt{2}}(|1\rangle + |2\rangle)$ of the above Harmonic oscilator.

   – What is expectation value of the energy?
   – Find $\langle x^2\rangle - \langle x\rangle^2$ at $t = 0$ in units of $\frac{\hbar}{m\omega}$.
   – Find $\langle x\rangle(t)$ (expectation at time $t > 0$) in units of $\sqrt{\frac{\hbar}{m\omega}}$.

3) Two quantum operators have the matrix representation

$$A = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad B = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \tag{1}$$

   a) A system is in quantum state $|\psi\rangle$ that is in an eigenfunction of operator A, corresponding to eigenvalue -1. Then for this state, what are $\langle A\rangle$ and ΔA?
   b) First A is measured and the result is $a = -1$. What is the state of the system after the measurement?
   c) Immediately afterwards, B is measured. What is the probability to find $b = 1$?
   d) Assuming that $b = 1$ was indeed found in (c), what is the state of the system after the measurement of B?

4) A particle in 1D is described by the usual Schroedinger equation with potential $V(x)$, which is a hybrid of the infinite well and the attractive Dirac-delta function, $V(x) = -\lambda\delta(x)$ for $|x| < L/2$ and $V(x) = +\infty$ for $|x| > L/2$. Usually we specify the parameters in H and ask for the ground state energy E_0, but this problem is backward: Assuming that the ground state energy E_0 is exactly zero, find the value of λ that makes that possible:

a) Consider the form of the Schrodeinger equation in the region $0 < x < L/2$ and find the general form of the solution. (Don't worry if the form looks slightly surprising.)

b) Of course, a similar form, with different coefficients, applies in $-L/2 < x < 0$. Use this knowledge to sketch the form of the wave function (which must be consistent with the boundary conditions).

c) Find the value of λ that solves the problem.