Homework 3, Quantum Mechanics 501, Rutgers

October 14, 2016

1) The normalized wave function $\psi(x,t)$ satisfies the time-dependent Schroedinger equation for a free particle of mass m moving in 1D. Consider a second wave function of the form $\phi(x,t) = exp(i(ax - bt))\psi(x - vt, t)$.

   – Show that $\phi(x,t)$ obeys the same time-dependent Schroedinger equation as $\psi(x,t)$ when constants a and b are choosen appropriately. What should the values of a and b be (express them in terms of v)?

   – Calculate the expectation value of position $\langle X \rangle$, momentum $\langle P \rangle$, and energy $\langle H \rangle$ for particle in the state $\phi(x,t)$ in terms of those for particle in the state $\psi(x,t)$. Show that uncertainty in the momentum is the same in both states.

   – What physical interpretation can be given to the transformation from the state $\psi(x,t)$ to the state $\phi(x,t)$?

2) A particle is in the ground state of a box of length L with infinitely high walls. Suddenly, the box expands (symmetrically) to length 2L, leaving the wave function momentarily undisturbed. Calculate the probability that measuring the energy of the system afterwards yields as result the ground state energy of the new box.

3) Consider the Gaussian wave packet of the form

$$\psi(x, t = 0) = \frac{1}{(\pi\Delta^2)^{1/4}} e^{ip_0 x/\hbar} e^{-\frac{x^2}{2\Delta^2}} \tag{1}$$

   Calculate the probability current j_x for every point x at time $t = 0$. Calculate explicitely the probability density, $P(x,t)$, at finite t using Hamiltonian of a free particle. Next, use this probability density to explicitely verify the validity of continuity equation at $t = 0$ ($\frac{\partial P(x,t)}{\partial t} = -\frac{\partial j(x,t)}{\partial x}$).

4) An atom of mass 4 10^9 eV/c^2 has its position measured within 2 nm accuracy. Assume that it is in a Gaussian wave packet state afterwards. How much time will elapse before the uncertainty of our knowledge about its position has doubled? How about a 1 g speck of matter that has been located to within 1 m?

5) A point-like particle of mass m sits in a one-dimensional potential well. The potential is infinitely high for $x < -s$ and for $x > +s$, while it is at a constant value of $V_0 > 0$ for $-s \le x < 0$ and zero for $0 \le x \le s$. The particle is in the ground state (lowest

energy eigenstate of the Hamiltonian) with energy $E_0 > V_0$.
Question: What is the probability that the particle can be found in the left half ($x < 0$) of the potential well? Outline how you would solve this problem step by step, without actually solving the (transcendental) equations that you encounter:

1.) Write down the one-dimensional Schrooedinger equation for this problem.
2.) Find the generic stationary solutions in the left and the right half of the potential well (you may assume $E > V_0$).
3.) List all boundary conditions that must be fulfilled (there are 4 of them!)
4.) Rewrite your two half-solutions from item 2. above to explicitly fulfill as many of the boundary conditions as possible.
5.) Outline how you would find the lowest energy (ground state eigenvalue E) that solves the one- dimensional Schrodinger equation. No closed algebraic solution is possible or required for this part - just explain which equation needs to be solved.
6.) Assuming you have E, how would you determine the normalization constants for the two half- solutions?
7.) Once you have those in hand as well, how can you answer the original question?