Homework 2, Quantum Mechanics 501, Rutgers

September 27, 2016

1) An operator **A**, corresponding to a physical observable, has two normalized eigenstates $|\phi_1\rangle$ and $|\phi_2\rangle$ with non-degenerate eigenvalues a_1 and a_2, respectively. A second operator **B**, corresponding to a different physical observable, has normalized eigenstates $|\chi_1\rangle$ and $|\chi_2\rangle$, with eigenvalues b_1 and b_2, respectively. The two sets of eigenstates are related by

$$|\phi_1\rangle \propto 2\,|\chi_1\rangle + 3\,|\chi_2\rangle \tag{1}$$
$$|\phi_2\rangle \propto 3\,|\chi_1\rangle - 2\,|\chi_2\rangle \tag{2}$$

The physical observable corresponding to **A** is meassured and the value a_1 is obtained. Immediately afterwards, the physical observable corresponding to **B** is measured, and again immediately after that the one corresponding to **A** is remeasured. What is the probability of obtaining a_1 a second time?

2) The ammonia molecule NH_3 has two different possible configurations: One (which we will call $|1\rangle$), where the nitrogen atom is located above the plane spanned by the three H atoms, and the other one (which we will call $|2\rangle$) where it is below. These two states span the Hilbert space in our simple example. In both states, the expectation value of the energy $\langle n|\,H\,|n\rangle$ is the same, $E(n = 1, 2)$. On the other hand, the two states are not eigenstates of the Hamiltonian; in fact, we have $\langle 2|\,H\,|1\rangle = \langle 1|\,H\,|2\rangle = -V$ (where V is some positive number).

   1) Write down the Hamiltonian in Dirac form and in matrix form.
   2) Find both eigenvalues and normalized eigenvectors. Which state is the ground state?
   3) Where is nitrogen in the two eigenstates, i.e., what is the probability to find nitrogen atom above or below in the two states?
   4) Consider the parity operator in which all coordinates change sign ($x \to -x$). Is parity well defined in the two eigenstates? If yes, what is the value of the parity operator in the two cases?

3) Consider the following three operators (representing physical observable) on the two

dimensional Hilbert space:

$$S_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \tag{3}$$

$$S_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \tag{4}$$

$$S_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{5}$$

(1) Assume S_z is measured and one finds the value -1. Immediately afterwards, what are $\langle S_x \rangle$, $\langle S_x^2 \rangle$ and $\Delta S_x \equiv \sqrt{\langle S_x^2 \rangle - \langle S_x \rangle^2}$

(2) What are the possible values one could measure for S_x, and what are their possibilities if it is measured immediately after measurement in (1).

(3) Explicitely calculate the commutators between any two of the three operators above (all 3). Is it possible to prepare a state of the system with well-defined values for all three?

4) Consider the following Hamiltonian for a classical system:

$$H = \frac{1}{2m}(p_x^2 + p_y^2 + p_z^2) + V(x^2 + y^2 + z^2) \tag{6}$$

Prove that the angular momentum is a constant of motion by explicitly evaluating Poisson bracket of say $L_z = xp_y - yp_x$ and H. Note that for such classical system $dL_z/dt = \{L_z, H\}$.