

Rutgers Physics 385 Electromagnetism I (Fall'11/Somalwar)

Solutions to Final Exam - Dec. 19, 2011

This is a closed book/notes exam. A one-sided 8.5x11 sheet with only formulae is allowed. Please attach the sheet to your solutions. Calculators are not needed. Indicate explicitly which problems should be graded. Indicate your reasoning clearly and step-by-step. Exam duration - Noon to 300PM. Total credit 40 points.

SECTION 1 - DO ANY 2 OUT OF 3 PROBLEMS, TOTAL 16 POINTS

A1. A sphere of linear dielectric material of radius R and dielectric constant k also has a uniform free charge density ρ . Find D,P and E inside and outside the sphere.

Solution: Total charge $Q = (4/3)\pi R^3\rho$. $D_{out} = Q/(4\pi r^2) = R^3\rho/3r^2$. $E_{out} = R^3\rho/3\epsilon_0r^2$. $P_{out} = 0$. Charge q enclosed in radius $r < R$ is $(4/3)\pi r^3\rho$. $D_{in} = q/(4\pi r^2) = r\rho/3$. $E_{in} = r\rho/3\epsilon_0k$. $P_{in} = \epsilon_0(k-1)E = ((k-1)/k)(r\rho/3)$.

A2. An infinitely large parallel plate capacitor has uniform surface charge density σ on its upper plate and $-\sigma$ on the lower plate. The plates are parallel to the xy plane, and are separated by a distance d . The entire capacitor is set moving with a constant speed v along the x axis. Find the magnetic field everywhere. (Hint: The surface current density for the positive plate would be $+\sigma v$ along the x -axis, for example. The rest is just Ampere's law.)

Solution: The outside field is zero. Using a vertical (in the yz plane) amperian loop of sides L that crosses the top plate (say), $BL = \mu_0 I_{enclosed} = \mu_0(\sigma v)L$. Hence, $B = \mu_0\sigma v$ along the y axis.

A3. A long straight wire of radius R carries a current density that rises linearly as a function of radius, $J(r) = \alpha r$. What is the total current? The wire is made up of a linear material with susceptibility χ_m . Use the appropriate versions of the Ampere law to find the magnetic fields H, B and M inside the wire.

Solution: Integrate the current density times the area element $2\pi r dr$ and get the enclosed current for $r < R$ to be $I(r) = 2\pi\alpha r^3/3$. (The total current then is $I = I(R) = 2\pi\alpha R^3/3$.) Therefore, $H(r) = I(r)/2\pi r = \alpha r^2/3$. $B(r) = \mu H = \mu_0(1 + \chi_m)\alpha r^2/3$. $M(r) = \chi_m H = \chi_m\alpha r^2/3$.

SECTION 2 - DO ANY 6 PROBLEMS, TOTAL 24 POINTS

B1. Find the energy stored in a thin spherical shell of radius R and charge Q by integrating E^2 and also by bringing in the total charge in increments of dq .

Solution: Integrate $\epsilon_0 E^2$ times the volume element $4\pi r^2 dr$ from R to infinity to get the energy to be $Q^2/8\pi\epsilon_0 R$. Alternatively, integrate the incremental energy $q dq/4\pi\epsilon_0 R$ from no charge to full charge Q to get the same answer.

B2. Find the capacitance per unit length of two coaxial metal cylindrical tubes (inner radius a and outer radius b), filled with a linear dielectric with dielectric constant k .

Solution: As usual, $E = q/(2\pi\epsilon_0 k r)$. Integrate from a to b to get the potential difference. Then $C = q/V = (2\pi\epsilon_0 k)/\ln(b/a)$.

B3. A uniform line charge λ is placed on an infinite straight wire, a distance d above a grounded conducting plane. Find the electric field as a function of distance from the wire ONLY in the plane formed by the wire and its image.

Solution: Use an image line charge the same distance on the other side. We know the field of an infinite line charge, so the field between the plane and the (real) line charge is $(\lambda/2\pi\epsilon_0)[1/r - 1/(2d - r)]$ and on the side away from the plane is $(\lambda/2\pi\epsilon_0)[1/r - 1/(2d + r)]$

B4. Find the potential everywhere for a spherical shell (radius R) that carries a constant surface charge density $\sigma(\theta) = k$ using Laplace's equation formalism. Make sure that the answers are the same as what you would get using Coulomb's law.

Solution: First with the Coulomb law: total charge $q = k(4\pi R^2)$. So the outside potential is $(q/4\pi\epsilon_0 r) = kR^2/\epsilon_0 r$. The inside potential ($r = R$) is kR/ϵ_0 . Now to Laplace. With the spherical symmetry, we know $l = 0$. That gives us for inside $A_0 r^0 P_0(\cos\theta) = A_0$ and for outside $B_0 r^{-1} P_0(\cos\theta) = B_0/r$. Matching at $r = R$ gives $B_0 = A_0 R$, but we still need to know A_0 . The charge density is the only remaining clue, so we match the perpendicular component of E , i.e. equate the difference in radial derivative of the potential on the surface with σ/ϵ_0 : $A_0/R = \sigma/\epsilon_0 = k/\epsilon_0$ to get $A_0 = kR/\epsilon_0$ and $B_0 = kR^2/\epsilon_0$, i.e., back to the Coulomb solutions.

B5. Compute the monopole and dipole terms for the potential V for a thin disk with radius

R and carrying a constant surface charge density σ . Assume the origin to be at the center of the disk.

Solution: The total charge is $\pi R^2\sigma$, so $V_{monopole} = \pi R^2\sigma/4\pi\epsilon_0 r = R^2\sigma/4\epsilon_0 r$. The x, y and z components of the dipole moment are zero, but one can also surmise that from the symmetry.

B6. Calculate the force of attraction per unit length between two infinitely long parallel wires, each carrying current I , and spaced a distance d apart.

Solution: As usual, $B = \mu_0 I/2\pi d$. Then $(F/L) = (qv/L)B = \lambda v B = \mu_0 I^2/2\pi d$.

B7. Evaluate the B field corresponding to the vector potential $\vec{A}_1 = z\hat{i}$. Then do the same for $\vec{A}_2 = (z\hat{i} - x\hat{k})/2$. What can you conclude about the difference of the two \vec{A} potentials? (Hint - evaluate the gradient of xz). Please be very specific.

Solution: The curls of both are just the unit vectors along y , so both vector potentials give the same B field. So their difference must be the gradient of a scalar function (which was conveniently given to you).

B8. Write down the boundary conditions for E and B (don't worry about the signs and constants). Attach the relevant Maxwell's equation to each boundary condition.

Solution: Change in perpendicular E is the charge density (divided by ϵ_0) from Gauss's law, change in parallel E is zero in statics because curl of E is zero and so on for the B field components.

B9. A square loop of wire (side a) lies in the xy plane centered at the origin, and carries a current I counterclockwise as viewed from the positive z axis. Calculate the magnetic dipole moment and the approximate \vec{A} at points far from the origin.

Solution: The magnetic dipole moment is area times current, so Ia^2 along the z axis and the dipole vector potential then is $(\mu_0/4\pi)Ia^2\sin(\theta)/r^2$ along ϕ .