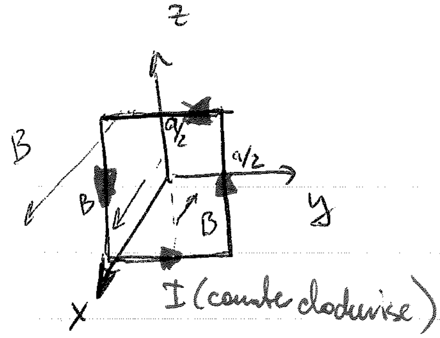
(1)



$$\vec{F} = I \int [d\vec{\ell} \times \vec{B}]$$

$$\vec{F}_{top} = I \cdot a \cdot k \cdot \frac{a}{2} \cdot (+\hat{z}) = +\frac{1}{2} k a^2 I \hat{z}$$

$$\vec{B} = k z \hat{x}$$

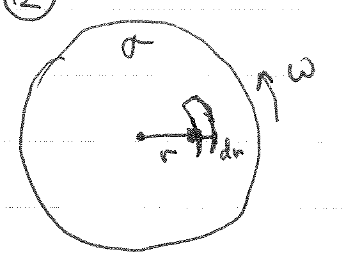
$$\vec{F}_{bottom} = I \cdot a \cdot k \left(-\frac{a}{2}\right) (-\hat{z}) = +\frac{1}{2} k a^2 I \hat{z}$$

$$\vec{F}_{right} = I \cdot \int_{-\frac{a}{2}}^{a/2} dz \cdot (kz) \cdot \hat{y} = 0$$

$$\vec{F}_{left} = I \cdot \int_{+\frac{a}{2}}^{-a/2} dz\, (kz)\, (-\hat{y}) = 0$$

$$\boxed{F_{total} = k a^2 I \cdot \hat{z}}$$

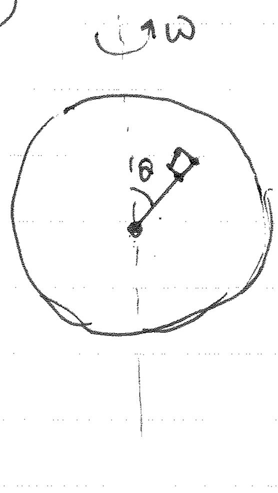
(2)



$d^2Q = dr \cdot \omega \cdot r \cdot dt \cdot \sigma \leftarrow$ total charge crossing dr in time dt

$$K = \frac{d^2Q}{dr\, dt} \Rightarrow \boxed{K = \sigma \cdot r \cdot \omega}$$

(3)



$dS = r \cdot d\theta \cdot dr$

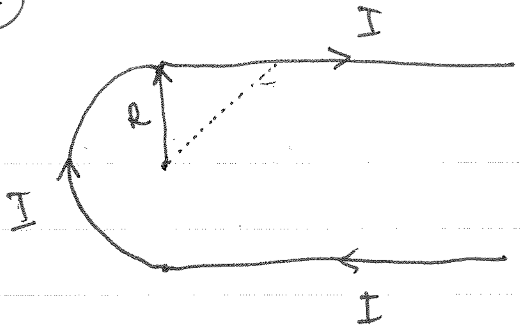
charge crossing dS in time dt

$dQ = \rho \cdot dS \cdot \omega \cdot r \cdot \sin\theta$

Current flows in direction $\hat{\varphi}$

$$\vec{J} = \rho \cdot \omega r \cdot \sin\theta \cdot \hat{\varphi}$$

(4)



$$\vec{B} = \frac{\mu_0}{4\pi}\int \frac{[\vec{I}\times\vec{r}]}{r^2}\,dl$$

$$\vec{B}_{top} = \frac{\mu_0}{4\pi}\cdot I\cdot(-\hat{z})\cdot\int_0^\infty \frac{dx}{R^2+x^2}\cdot\frac{R}{\sqrt{R^2+x^2}}$$

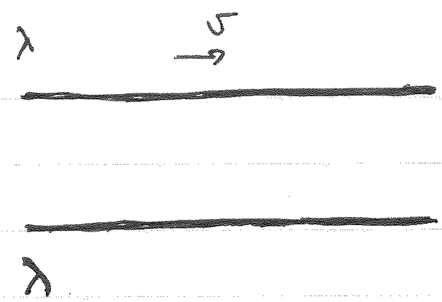
$$\vec{B}_{bot} = \frac{\mu_0}{4\pi}\, I\,(-\hat{z})\cdot\int_0^\infty \frac{dx}{R^2+x^2}\cdot\frac{R}{\sqrt{R^2+x^2}}$$

$$\vec{B}_{side} = \frac{\mu_0}{4\pi}\cdot I\,(-\hat{z})\cdot\frac{\pi R}{R^2}$$

$$\int_0^\infty \frac{dx}{(R^2+x^2)^{3/2}} = \frac{1}{R^2}\int_0^\infty \frac{du}{(1+u^2)^{3/2}} = \frac{1}{R^2}$$

$$\vec{B} = -\hat{z}\cdot\frac{\mu_0}{4\pi}\, I\left(\frac{1}{R}+\frac{1}{R}+\frac{\pi}{R}\right) = -\hat{z}\,\frac{\mu_0 I}{4R}\left(\frac{2}{\pi}+1\right)$$

(5)



Electric force (per unit length)

$$E = \frac{\lambda}{2\pi\varepsilon_0 \cdot d}, \quad f_e = \frac{\lambda^2}{2\pi\varepsilon_0 \cdot d} = \frac{\lambda^2}{2\pi \cdot d} \cdot \frac{1}{\varepsilon_0}$$

Magnetic force: $I = v \cdot \lambda$

$$f_m = \frac{\mu_0}{2\pi} \frac{I^2}{d} = \frac{\mu_0 \lambda^2 v^2}{2\pi d} = \frac{\lambda^2}{2\pi d} \cdot \mu_0 v^2$$

$$\frac{f_m}{f_e} = \varepsilon_0 \mu_0 v^2$$

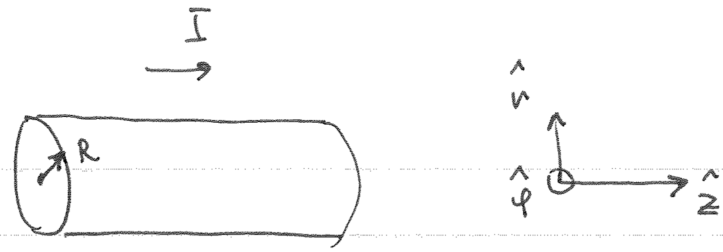
$\sqrt{\varepsilon_0 \mu_0} = \frac{1}{c}$, where c is the speed of light,

so

$$\frac{f_m}{f_e} = \frac{v^2}{c^2}$$ $\leftarrow$ equal at $v = c$

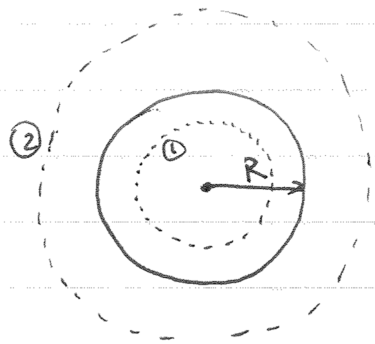
not a reasonable speed

6 -1-



a) only surface current!

cylindrical symmetry: B only has $\hat{\varphi}$ component



1 inside the wire:

$$B \cdot 2\pi r = \mu_0 \cdot 0 \Rightarrow \boxed{B = 0\,, \quad r < R}$$

2 outside the wire:

$$B \cdot 2\pi r = \mu_0 I \Rightarrow \boxed{B = \frac{\mu_0 I}{2\pi r}\,, \quad r > R}$$

(same as infinite thin wire!)

B jumps ~~at~~ at $R = r$

$$\Delta B = \frac{\mu_0 I}{2\pi R}$$

Boundary condition: $\Delta B = \mu_0 K$

in this case $K = \dfrac{I}{2\pi R}$, so

$$\Delta B = \frac{\mu_0 I}{2\pi R}\,,$$ as it should be

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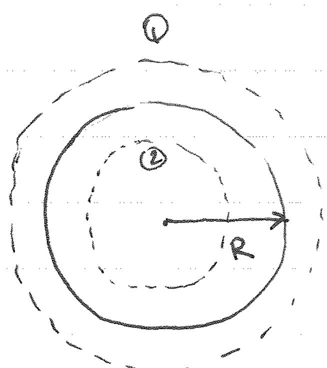
(6) -2-

b) volume current: $\vec{J} = \hat{z} \cdot k \cdot r$

let's determine k:

$$\iint r\,d\varphi \cdot dr \cdot J = I \Rightarrow I = k \cdot 2\pi \int_0^R r^2 dr = k \cdot 2\pi \frac{R^3}{3}$$

$$\boxed{k = I \cdot \frac{3}{2\pi R^3}} \qquad J = \frac{3I}{2\pi R^3} \cdot r$$



(1) outside the wire:

$$B \cdot 2\pi r = \mu_0 I$$

$$\boxed{B = \frac{\mu_0 I}{2\pi r}, \quad r > R} \leftarrow \text{same as thin wire}$$

(2) inside:

$$B \cdot 2\pi r = \mu_0 \int_0^r \int_0^{2\pi} J \cdot r\,dr\,d\varphi = 2\pi \mu_0 \cdot \frac{3I}{2\pi R^3} \cdot \int_0^r r^2 dr =$$

$$= \cancel{2\pi} \mu_0 \cdot \frac{\cancel{3} I}{\cancel{2\pi} R^3} \cdot \frac{r^3}{\cancel{3}} = \mu_0 I \cdot \frac{r^3}{R^3}$$

$$\boxed{B = \frac{\mu_0 I}{2\pi} \frac{r^2}{R^3}}$$