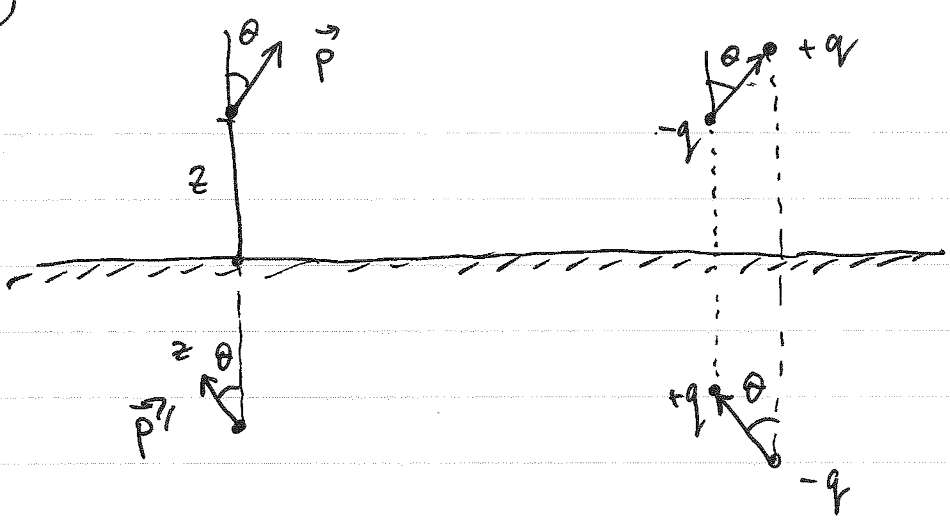
(1)



Dipole "reflets" in the conducting plane not like a usual vector (that's why $\vec{p}$ is pseudovector, unlike true vectors like speed or force)

$\vec{\tau} = \vec{p} \times \vec{E}$, where $\vec{E}$ is the field of $\vec{p}'$

$$\vec{\tau} = \vec{p} \times \frac{1}{4\pi\varepsilon_0 r^3}\left[-\vec{p}' + 3(\vec{p}'\cdot\hat{n})\hat{n}\right] =$$

$$= \frac{-1}{4\pi\varepsilon_0 (2z)^3}\left[[\vec{p}\times\vec{p}'] - 3\cdot p\cdot\cos\theta\cdot[\vec{p}\times\hat{n}]\right] = ..$$

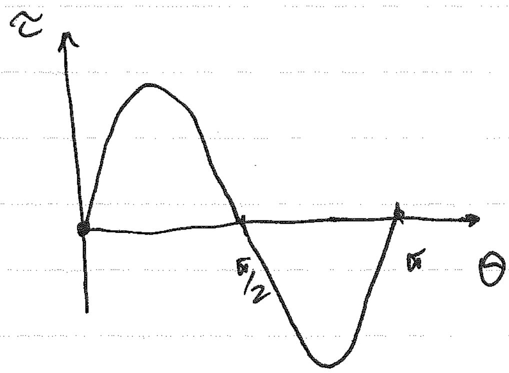
both terms $[\vec{p}\times\vec{p}']$ and $[\vec{p}\times\hat{n}]$ point counterclockwise let's set this direction as positive

$$|\tau| = \frac{-1}{4\pi\varepsilon_0 (2z)^2}\left[p^2\cdot\sin 2\theta - 3p^2\cos\theta\sin\theta\right] =$$

$$= \frac{-p^2}{4\pi\varepsilon_0 (2z)^2}\left[\sin 2\theta - \frac{3}{2}\sin 2\theta\right] = \dots$$

① cont'd

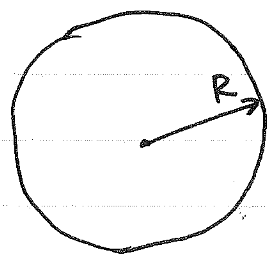
$$\tau = \frac{p^2}{8\pi \varepsilon_0 (2z)^3} \cdot \sin 2\theta \rightarrow \text{counterclockwize.}$$



← if θ is less then $\frac{\pi}{2}$ it will rotate up, otherwise it will rotate down

↑ and ↓ are stable equilibria

2



$\vec{P}(r) = k\vec{r}$

$\rho_b = -\vec{\nabla}\vec{P} = -\frac{1}{r^2}\frac{\partial}{\partial r}(r^2 k r) = \underline{\underline{-3k}}$

$\sigma_b = \vec{P}\cdot\hat{n} = kR\,\hat{r}\,\hat{n} = \underline{\underline{kR}}$

Field outside: same as point charge.

$$\left.\begin{aligned}\int_{\text{sphere}} \rho_b \, d\tau &= -3k\cdot\frac{4}{3}\pi R^3 = -4\pi R^3 \cdot k\\ \int_{\text{surface}} \sigma_b \, dS &= kR\cdot 4\pi R^2 = +4\pi R^3 k\end{aligned}\right\} Q_{tot} = 0$$

field outside is $\underline{\text{zero}}$

<u>Inside the sphere:</u>

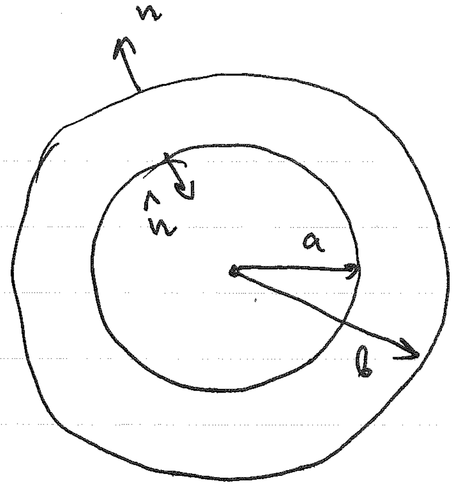
Replace dielectric with ρ_b and σ_b → σ_b does not matter

$4\pi r^2 E = \frac{4}{3}\pi r^3(-3k)\frac{1}{\varepsilon_0}$ ← Gauss law

$E = -\frac{k}{\varepsilon_0} r$

$$\boxed{\vec{E} = -\frac{k}{\varepsilon_0}\vec{r}}$$

③



$$\vec{P}(\vec{r}) = \frac{k}{r}\hat{r}$$

a) $$\rho_b = -\vec{\nabla}\cdot\vec{P} = -\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{k}{r}\right) =$$

$$= -\frac{1}{r^2}\frac{d}{dr}\left(+kr\right) = \boxed{-\frac{k}{r^2} = \rho_b}$$

$$\sigma_b = \hat{P}\cdot\hat{n} = \begin{cases} \text{outer surface: } \frac{k}{b}\,\hat{r}\cdot\hat{n} = +\frac{k}{b} \\ \text{inner surface: } \frac{k}{a}\cdot\hat{r}\cdot\hat{n} = -\frac{k}{a} \end{cases}$$

$$E = \begin{cases} 0 & r < a \\ 0 & r > b \\ \text{for} & a < b < r: \end{cases}$$

← total enclosed charge is zero

$$4\pi r^2 E = \frac{1}{\varepsilon_0}\left[\int_a^r \rho_b(r)\cdot d\tau + \oint_{\substack{\text{inner}\\ \text{surf}}} \sigma_b^{\text{inner}}\cdot ds\right]$$

$$\oint_{\substack{\text{inner}\\ \text{surf}}} \sigma_b^{\text{inner}}\, dS = -\frac{k}{a}\cdot 4\pi a^2 = -4\pi k\cdot a$$

$$\int_a^r \rho_b(r)\,d\tau = 4\pi\int_a^r\left(-\frac{k}{r^2}\right)\cdot r^2 dr = -4\pi k\int_a^r dr = -4\pi k(r-a)$$

$$4\pi r^2 E = \frac{1}{\varepsilon_0}\left(-\cancel{4\pi k a} - 4\pi k r + \cancel{4\pi k a}\right) = -\frac{4\pi k\cdot r}{\varepsilon_0}$$

$$\boxed{\vec{E} = -\frac{k}{\varepsilon_0 r}\hat{r}}, \text{ for } a < r < b.$$

③ cont

b) Now, use $\vec{D}$:

$\oint D\,dS = Q_f$ → no free charges!

$D = 0$ everywhere!

$E = D$ for $r < a$ and $r > b$, $E = 0$ there

for $a < r < b$:

$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P} \Rightarrow \underline{\vec{E} = -\frac{\vec{P}}{\varepsilon_0} = -\frac{k\,\hat{r}}{\varepsilon_0 r}},$$

same answer as in a) but much simpler!