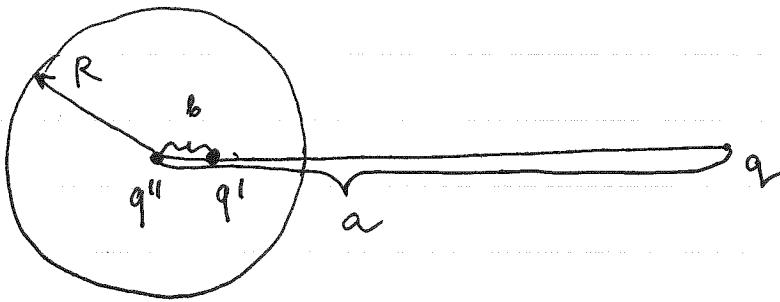


(1)



For grounded sphere, we have

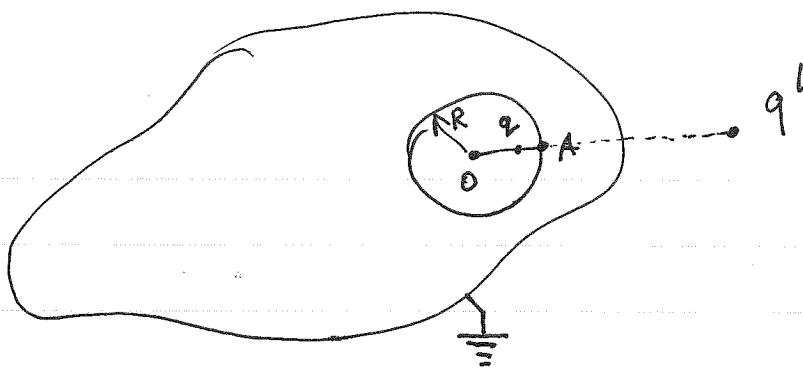
$$V_{\text{sphere}} = 0 \text{ and } q' = -\frac{R}{a}q \text{ at } b = \frac{R^2}{a} \text{ from center.}$$

The charge that we need to add only needs to change the potential at the surface of the sphere by a constant — and one way to do it is to place the charge at the center.

For uncharged sphere, $q'' = -q'$, and the force is

$$\begin{aligned} |F| &= \frac{1}{4\pi\epsilon_0} \left[\frac{q''q}{a^2} + \frac{q'q}{(a-b)^2} \right] = \frac{q^2}{4\pi\epsilon_0} \frac{R}{a} \left[\frac{1}{(a-b)^2} - \frac{1}{a^2} \right] = \\ &= \frac{q^2}{4\pi\epsilon_0} \frac{R}{a} \left[\frac{1}{(a-\frac{R^2}{a})^2} - \frac{1}{a^2} \right] = \boxed{\frac{q^2}{4\pi\epsilon_0} \frac{R}{a} \left[\frac{a^2}{(a^2-R^2)^2} - \frac{1}{a^2} \right]} \end{aligned}$$

(2)



This is image problem inside-out:
we need potential inside the hollow, and the image, therefore, will be outside of the hollow - it could be inside or outside the conductor - that does not matter: Laplace's eqn does not care what is happening outside the volume it is solved in, as long as boundary conditions are specified.

So, using example from class (and/or previous problem): $q' \rightarrow q$

$$b \rightarrow |OA| = d$$

$$R \rightarrow R$$

need to find a' and q' , so

$$q' = -\frac{a}{R}q, \quad d = \frac{R^2}{a} \Rightarrow$$

$$\boxed{\begin{aligned} a &= \frac{R^2}{d} \\ q' &= -\frac{R}{d}q \end{aligned}}$$

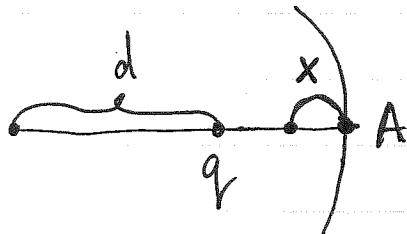
Potential inside is the sum of potentials of q and q' . At point A,

$$V = \frac{1}{4\pi\epsilon_0} \left(\frac{q}{(R-d)} + \frac{q'}{(a-R)} \right) = \dots$$

② cont'd

$$= \frac{q}{4\pi\epsilon_0} \left[\frac{1}{R-d} + \frac{-R/d}{\frac{R^2}{d} - R} \right] = 0, \quad (\text{as it should!})$$

$\sigma = -\epsilon_0 \frac{\partial V}{\partial n}$, where \hat{n} points towards center of the sphere.



$$V(x) = \frac{q}{4\pi\epsilon_0} \left[\frac{1}{R-d-x} + \frac{R/d}{a-R+x} \right]$$

$$\frac{\partial V}{\partial x} = \frac{q}{4\pi\epsilon_0} \left[\frac{1}{(R-d-x)^2} + \frac{R/d}{(a-R+x)^2} \right]$$

$$\sigma = -\epsilon_0 \frac{\partial V}{\partial x} \Big|_{x=0} = -\frac{q}{4\pi} \left[\frac{1}{(R-d)^2} + \frac{R/d}{\left(\frac{R^2}{d} - R\right)^2} \right] =$$

$$= -\frac{q}{4\pi} \left[\frac{1}{(R-d)^2} + \frac{R/d}{\frac{1}{d^2}(R^2-Rd)^2} \right] = -\frac{q}{4\pi} \left[\frac{1}{(R-d)^2} + \frac{d}{R(R-d)^2} \right] =$$

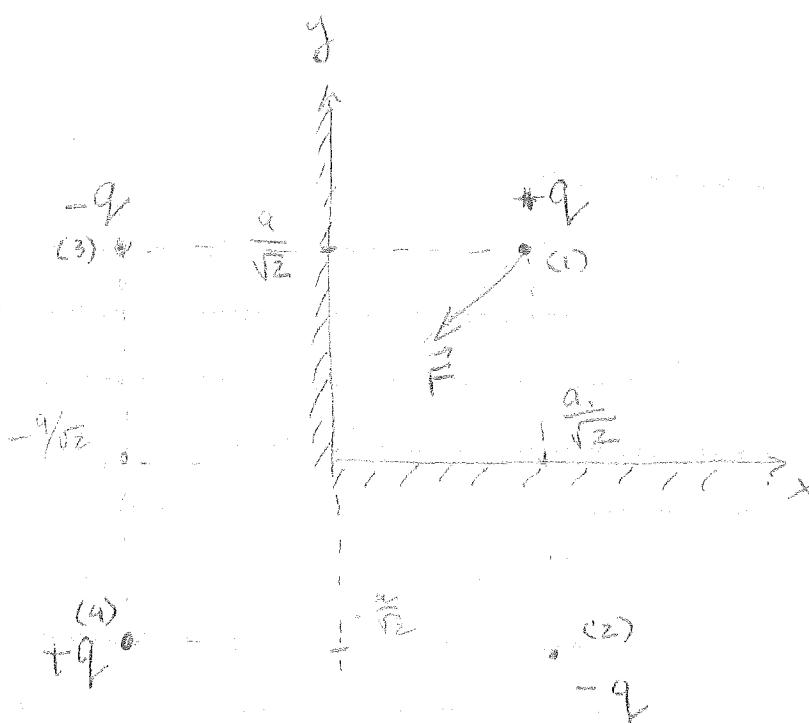
$$\sigma = \frac{q}{4\pi(R-d)^2} \left(1 + \frac{d}{R} \right)$$

if $d=0 \rightarrow$ uniform charge, $q/4\pi R^2 \rightarrow$ as we could guess

if $d \rightarrow R \rightarrow$ similar singularity as with charge above infinite conducting plane

(see textbook: $\sigma(0,0) = -\frac{q}{2\pi d^2}$, where d is distance above the plane)

(3)



Boundary condition:

$$V(0, y) = V(x, 0) = 0$$

To satisfy $V(0, y) = 0$ we can add $-q$ at $(\frac{a}{\sqrt{2}}, -\frac{a}{\sqrt{2}})$

To satisfy $V(x, 0) = 0$ we can add $-q$ at $(-\frac{a}{\sqrt{2}}, \frac{a}{\sqrt{2}})$

These two imaginary charges and the original charge will not satisfy the boundary conditions → need the third imaginary charge

If we put $+q$ at $(-\frac{a}{\sqrt{2}}, -\frac{a}{\sqrt{2}})$

$V(0, y)$ will be satisfied because charge (1) is balanced by (2) and (3) by (4).

$V(x, 0)$ is satisfied because (1) is balanced by (3) and (2) by (4).

So: F will point along the bisector, into the corner.

$$|F| = \frac{q^2}{4\pi\epsilon_0} \left(\frac{1}{(\sqrt{2})^2} + \frac{1}{(2a)^2} \right) = \frac{q^2}{4\pi\epsilon_0} \frac{2\sqrt{2}+1}{4a^2} = \cancel{\frac{q^2}{4\pi\epsilon_0} \frac{2\sqrt{2}+1}{4a^2}}$$