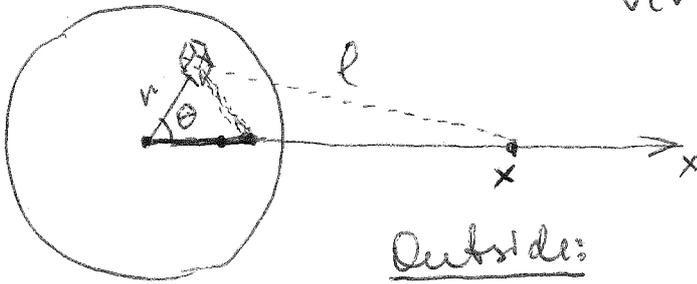


①

 $V(r) \rightarrow$ no ϕ, θ dependence.Outside:

$$dq = \rho \cdot r^2 \sin\theta \cdot dr d\theta d\phi$$

$$dV = \frac{1}{4\pi\epsilon_0} \frac{dq}{l}$$

$$l = \sqrt{x^2 + r^2 - 2xr \cos\theta}$$

$$V(x) = \frac{\rho}{4\pi\epsilon_0} \cdot 2\pi \int_0^R \int_0^\pi \frac{\sin\theta d\theta}{\sqrt{x^2 + r^2 - 2xr \cos\theta}} \cdot r^2 dr$$

$$\int \frac{\sin\theta d\theta}{\sqrt{x^2 + r^2 - 2xr \cos\theta}} = - \int \frac{du}{\sqrt{x^2 + r^2 - 2xr u}} = - \frac{-2xr}{-2xr} \int \frac{du}{\sqrt{x^2 + r^2 - 2xr u}}$$

$(u = \cos\theta)$

$$= \frac{1}{2xr} \int \frac{d(x^2 + r^2 - 2xr u)}{\sqrt{x^2 + r^2 - 2xr u}} = \frac{1}{xr} \cdot \sqrt{x^2 + r^2 - 2xr \cos\theta} + C$$

$$\int_0^\pi (\) = \frac{1}{xr} \left(\sqrt{x^2 + r^2 + 2xr} - \sqrt{x^2 + r^2 - 2xr} \right) =$$

$$= \frac{1}{xr} \left(x+r - \sqrt{(x-r)^2} \right) =$$

$$= \frac{1}{xr} (x+r - x+r) = \frac{2}{x}$$

here,
assume
 $x > r$
(outside!)

①
page 2

$$V(x) \Big|_{x>R} = \frac{\rho}{4\pi\epsilon_0} 2\pi \int_0^R \frac{2}{x} r^2 dr = \frac{\rho}{\epsilon_0} \frac{1}{x} \frac{1}{3} R^3$$

q of sphere: $q = \rho \cdot \frac{4}{3}\pi R^3 \Rightarrow \frac{1}{3}\rho R^3 = \frac{q}{4\pi}$

$$V(x) \Big|_{x>R} = \frac{q}{4\pi\epsilon_0 x} \rightarrow$$

$$V(R) = \frac{q}{4\pi\epsilon_0 R} = \frac{\rho R^2}{3\epsilon_0}$$

for $x < R$:

two regions: $x < r$ and $x > r$

$$\int_0^r (\) = \begin{cases} \frac{2}{r} & \text{if } x < r \\ \frac{2}{x} & \text{if } x > r \end{cases}$$

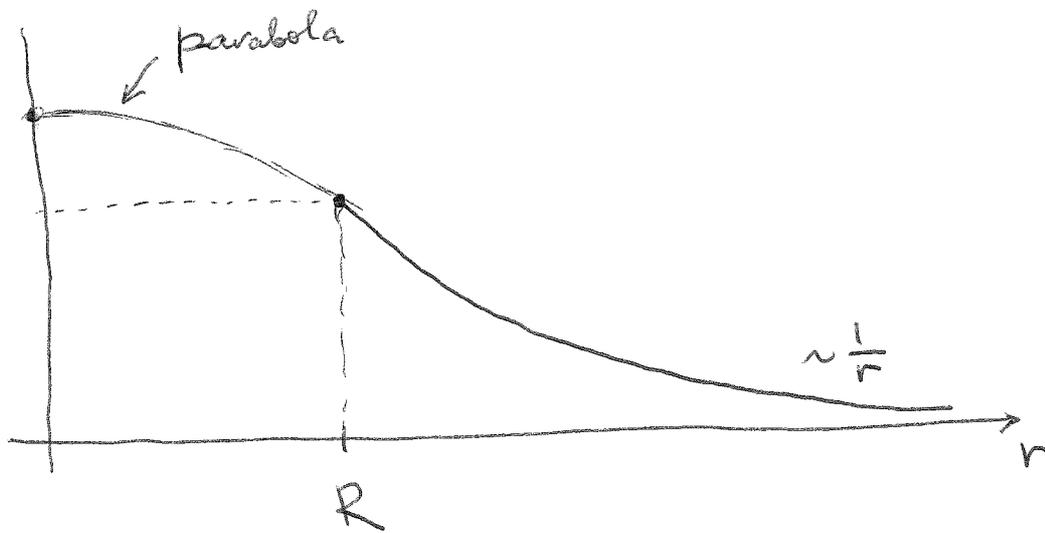
$$V(x) = \frac{\rho}{4\pi\epsilon_0} 2\pi \int_0^R \left(\int_0^r (\) \right) \cdot r^2 dr = \frac{\rho}{2\epsilon_0} \left[\int_0^x \frac{2}{x} r^2 dr + \int_x^R \frac{2r^2}{r} dr \right]$$

$$= \frac{\rho}{2\epsilon_0} \left[\frac{2}{x} \cdot \frac{1}{3} x^3 + 2 \int_x^R r dr \right] = \frac{\rho}{\epsilon_0} \left[\frac{1}{3} x^2 + \frac{1}{2} R^2 - \frac{1}{2} x^2 \right] =$$

$$V(x) \Big|_{x < R} = \frac{\rho}{\epsilon_0} \left[\frac{1}{2} R^2 - \frac{1}{6} x^2 \right]$$

$$V(R) = \frac{\rho}{\epsilon_0} \cdot \frac{R^2}{3}$$

continuous, ~~no~~
as it should!



$-\vec{\nabla}V$: in spherical coordinates: $\vec{\nabla}V = \frac{\partial V}{\partial r} \hat{r} + 0$ ($V_\varphi = 0$
 $V_\theta = 0$)

$r < R$:

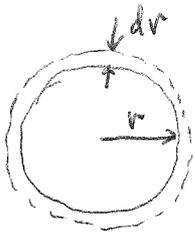
$$\vec{E} = -\vec{\nabla}V = -\hat{r} \cdot \frac{\rho}{\epsilon_0} \cdot \left(-\frac{1}{6} \cdot 2r\right) = \frac{\rho}{3\epsilon_0} r \cdot \hat{r}$$

$r > R$:

$$\vec{E} = -\vec{\nabla}V = -\hat{r} \cdot \frac{\rho}{3\epsilon_0} \cdot R^3 \cdot \left(-\frac{1}{r^2}\right) = \frac{\rho}{3\epsilon_0} \frac{R^3}{r^2} \hat{r}$$

that coincides with the answer to problem (3) in HW 2.

②



$$dq = 4\pi r^2 \cdot dr \cdot \rho$$

Work to bring that charge dq to the sphere of radius r and charge

$$Q = \frac{4}{3}\pi r^3 \cdot \rho \text{ is } dW = dq \cdot V,$$

where V is potential on the surface of the sphere. From previous problem, it is

$$V = \frac{\rho r^2}{3\epsilon_0}$$

$$dW = 4\pi r^2 \cdot \rho \cdot dr \cdot \frac{\rho \cdot r^2}{3\epsilon_0} = \frac{4\pi \cdot \rho^2}{3\epsilon_0} r^4 \cdot dr$$

$$W = \int_0^R dW = \frac{4\pi \rho^2}{15\epsilon_0} \cdot R^5$$

(3)

$$W = \frac{\epsilon_0}{2} \int_{\text{all space}} E^2 dV$$

$$E = \frac{q \cdot r}{3\epsilon_0} \quad (r < R)$$

$$E = \frac{q R^3}{3\epsilon_0} \frac{1}{r^2} \quad (r > R)$$

$$W = \frac{\epsilon_0}{2} \int_0^\infty r^2 dr \int_0^\pi \sin\theta d\theta \int_0^{2\pi} d\phi \cdot E^2 =$$

$$= \frac{\epsilon_0}{2} \cdot 2 \cdot 2\pi \cdot \int_0^\infty E^2 \cdot r^2 dr = 2\pi\epsilon_0 \left(\int_0^R \left(\frac{q r}{3\epsilon_0}\right)^2 r^2 dr + \right.$$

$$\left. + \int_R^\infty \left(\frac{q R^3}{3\epsilon_0} \cdot \frac{1}{r^2}\right)^2 r^2 dr \right) = 2\pi\epsilon_0 \cdot \frac{q^2}{9\epsilon_0^2} \left(\int_0^R r^4 dr + R^6 \int_R^\infty \frac{dr}{r^2} \right)$$

$$= \frac{2\pi \cdot q^2}{9\epsilon_0} \left(\frac{1}{5} R^5 + R^6 \left(-\frac{1}{r} \right) \Big|_R^\infty \right) = \frac{2\pi q^2}{9\epsilon_0} R^5 \left(\frac{1}{5} + 1 \right) =$$

$$= \frac{q^2 R^5}{\epsilon_0} \cdot \frac{2\pi}{9} \cdot \frac{6}{5} = \frac{4\pi q^2}{15} R^5$$