① $\vec{E} = k r^3 \hat{r}$

a) ρ - ? $\quad \vec{\nabla}\vec{E} = \frac{1}{\varepsilon_0}\rho \qquad \rho = \varepsilon_0 \vec{\nabla}\vec{E}$

$$\vec{\nabla}\vec{E} = \frac{1}{r^2}\frac{\partial(E_r \cdot r^2)}{\partial r} + \frac{1}{r}\frac{\partial E_\theta}{\partial \theta} + \frac{1}{r \cdot \sin\varphi}\frac{\partial(\sin\varphi \cdot E_\varphi)}{\partial \varphi}$$

$$E_r = \vec{E}\hat{r} = kr^3 ; \quad E_\theta = \vec{E}\hat{\theta} = 0 ; \quad E_\varphi = \vec{E}\hat{\varphi} = 0 .$$

$$\vec{\nabla}\vec{E} = \frac{1}{r^2}\frac{\partial(k \cdot r^5)}{\partial r} = \frac{5k}{r^2}\cdot r^4 = 5k \cdot r^2$$

$$\underline{\rho = 5k\varepsilon_0 \cdot r^2}$$

b) $Q(R)$

1) ~~[illegible]~~ integrate in spherical coordinates:

$$Q = \int_0^{2\pi} d\varphi \int_0^{\pi} d\theta \int_0^R dr \cdot r^2 \sin\theta \cdot 5k\varepsilon_0 \cdot r^2 =$$

$$2\pi \cdot 5k\varepsilon_0 \cdot \int_0^{\pi} \sin\theta \, d\theta \int_0^R r^4 dr = 2\pi \cdot \cancel{5}k\varepsilon_0 \cdot 2 \cdot \cancel{\tfrac{1}{5}} R^5 =$$

$$\underline{Q = 4\pi k \varepsilon_0 R^5}$$

2) short cut in spherial coordinats:

volume of spherial shell of R, $R+\Delta R$ is $4\pi R^2 \cdot \Delta R$

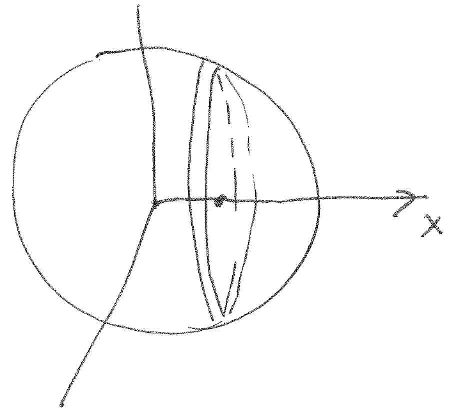
$$Q = \int_0^R 4\pi r^2 dr \cdot 5k\varepsilon_0 r^2 = 4\pi k\varepsilon_0 \, 5 \int_0^R r^4 dr = \underline{4\pi k\varepsilon_0 \cdot R^5}$$

3) Gauss law: $Q = \varepsilon_0 \oint \vec{E}\, d\vec{s}$, use sphere radius R:

$$Q = \varepsilon_0 \cdot 4\pi R^2 \cdot kR^3 = \boxed{4\pi k\varepsilon_0 R^5 = Q}$$

1

b4) Cartesian coordinates:



slice in x

slice radius $L = \sqrt{R^2 - x^2}$

$$Q = \int_{-R}^{R} \left(\S_{slice} dS \right) \cdot dx$$

$$\S_{slice} = \iint 5k\varepsilon_0 r^2 \cdot dS = \int_0^L \int_0^{2\pi} 5k\varepsilon_0 (x^2 + \ell^2)\, \overbrace{\ell \cdot d\ell \cdot d\varphi}^{dS} =$$

$$= 5k\varepsilon_0 \cdot 2\pi \cdot \int_0^L (x^2 \ell + \ell^3)\, d\ell = 10k\varepsilon_0 \cdot \pi \cdot \left(x^2 \frac{L^2}{2} + \frac{L^4}{4} \right) =$$

$$= \frac{5\pi k\varepsilon_0}{2} L^2 (2x^2 + L^2) = \frac{5\pi k\varepsilon_0}{2} (R^2 - x^2)(2x^2 + R^2 - x^2) =$$

$$= \frac{5\pi k\varepsilon_0}{2} (R^2 - x^2)(R^2 + x^2) = \frac{5\pi k\varepsilon_0}{2} (R^4 - x^4)$$

$$Q = \int_{-R}^{R} \frac{5\pi k\varepsilon_0}{2} (R^4 - x^4)\, dx = \frac{5\pi k\varepsilon_0}{2} R^4 \int_{-R}^{R} dx - \frac{5\pi k\varepsilon_0}{2} \int_{-R}^{R} x^4 dx =$$

$$= \frac{5\pi k\varepsilon_0}{2} \left(R^4 \cdot (R + R) - \frac{1}{5}(R^5 + R^5) \right) = \frac{\cancel{5}\pi k\varepsilon_0}{\cancel{2}} \frac{\cancel{2}}{\cancel{5}} (5R^5 - R^5) =$$

$$= \underline{4\pi k\varepsilon_0 \cdot R^5}$$

(1)

c) $\vec{\nabla}\vec{E}$: spherical: $5kr^2$ (see a))

cartesian:

$$\vec{E} = kr^3\hat{r} \Rightarrow E_x = kr^3 \cdot \frac{x}{r} = kr^2 \cdot x = k \cdot x(x^2+y^2+z^2)$$

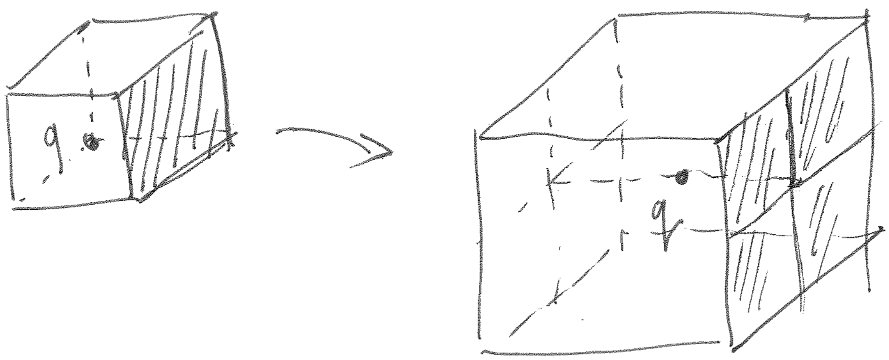
$$E_x = kx^3 + kx(y^2+z^2)$$
$$E_y = ky^3 + ky(x^2+z^2)$$
$$E_z = kz^3 + kz(x^2+y^2)$$

$$\vec{\nabla}\vec{E} = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} = 3kx^2 + k(y^2+z^2) +$$
$$+ 3ky^2 + k(x^2+z^2) +$$
$$+ 3kz^2 + k(x^2+y^2) =$$

$$= x^2(3k+k+k) + y^2(3k+k+k) + z^2(3k+k+k) =$$

$$\boxed{\vec{\nabla}\vec{E} = 5k(x^2+y^2+z^2) = \underline{\underline{5k \cdot r^2}}}$$

②



Total flux throug the large cube is $\frac{q}{\varepsilon_0}$

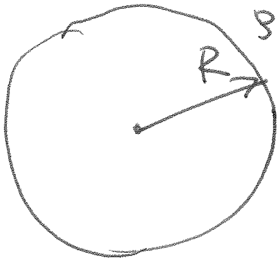
~~Flux~~ Flux throu one of the six faces = $\frac{q}{6\varepsilon_0}$

what we have is $\frac{1}{4}$ of 1 face →

$$\boxed{\Phi = \frac{1}{24}\frac{q}{\varepsilon_0}}$$

(3)

Gauss law:

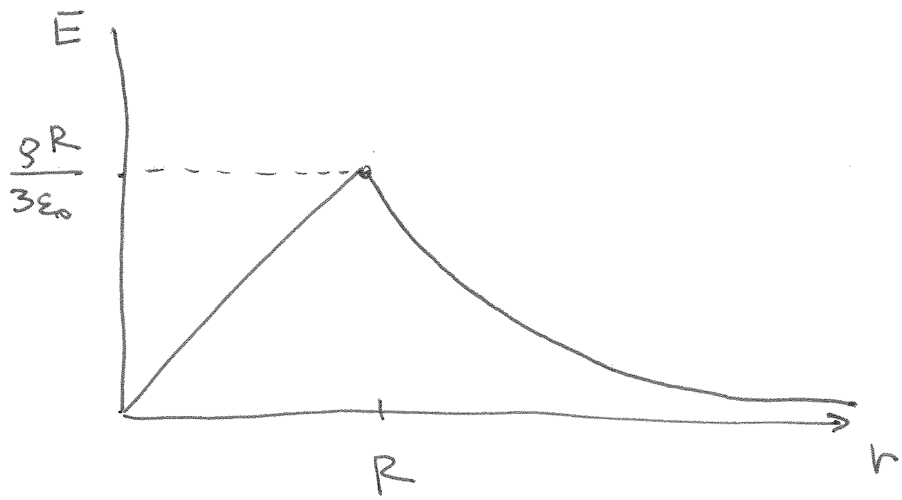


$r > R, \quad E = \frac{1}{4\pi\varepsilon_0} \frac{Q}{r^2}$

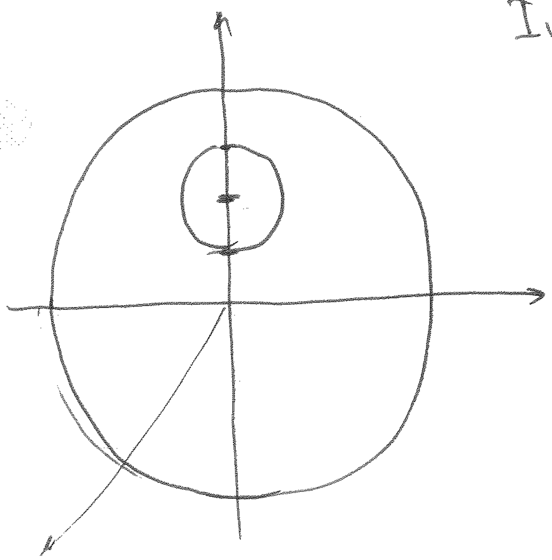
$Q = \frac{4}{3}\pi R^3 \cdot \rho, \quad \boxed{E = \frac{\rho R^3}{3\varepsilon_0} \frac{1}{r^2}}$

$r < R$

$Q = \frac{4}{3}\pi r^3 \cdot \rho, \quad E = \frac{\rho r^3}{3\varepsilon_0} \frac{1}{r^2} = \frac{\rho}{3\varepsilon_0} \cdot r$



④



Instead of hollowed sphere:

Two spheres:

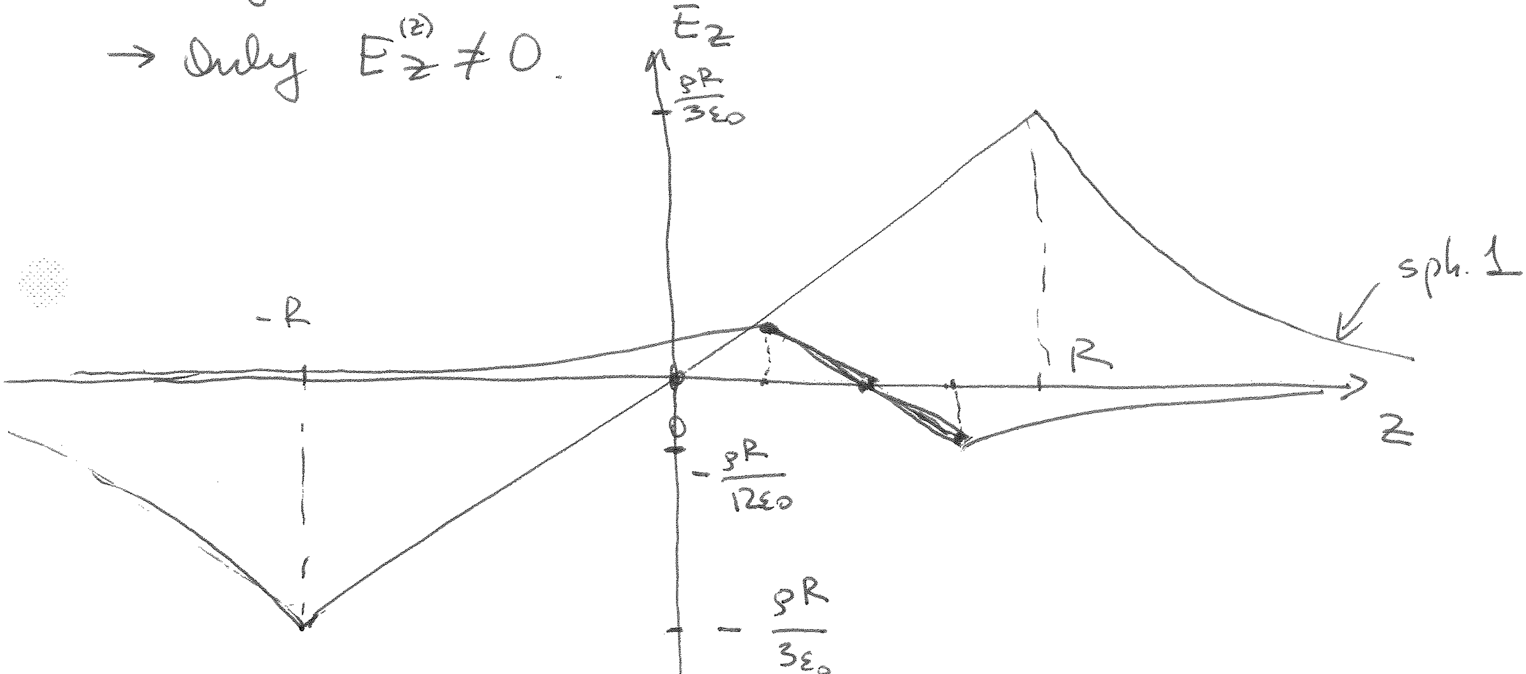
$+\rho$, $(0,0,0)$, radius R, $Q = \frac{4}{3}\pi R^3 \rho = Q_0$

$-\rho$, $(0,0,\frac{R}{2})$, radius $R/4$, $Q = \frac{Q_0}{64}$

Use superposition principle:

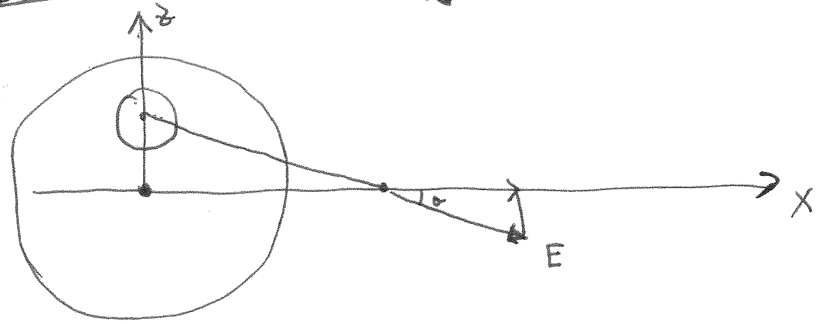
Along z axis: $r = (0,0,z)$

$\rightarrow$ Only $E_z^{(z)} \neq 0$.



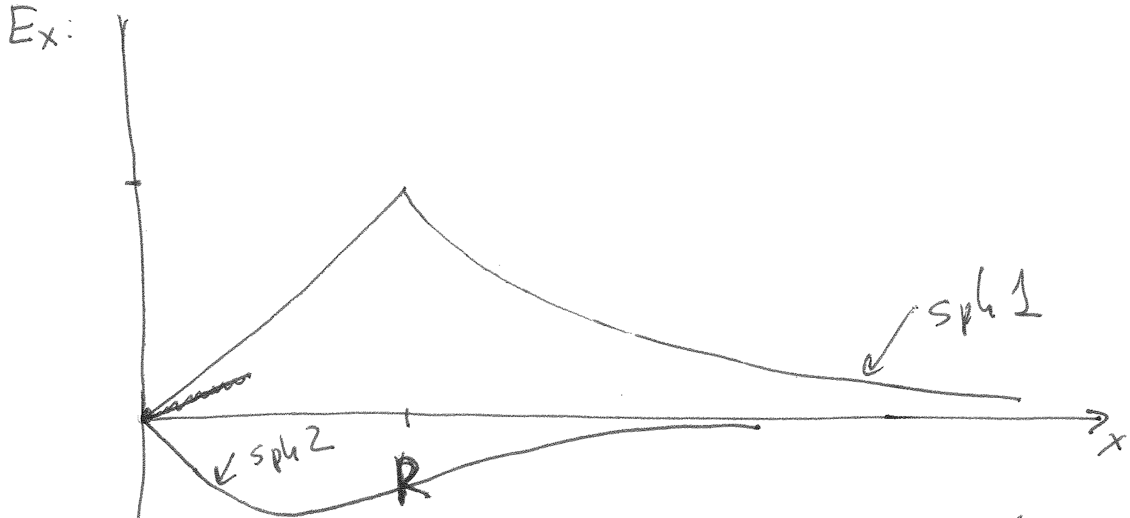
$$E_z^{(z)} = \begin{cases} z < -R: & -\frac{Q_0}{4\pi\varepsilon_0}\cdot\left(\frac{1}{z^2} - \frac{1/64}{(z-\frac{R}{2})^2}\right) = -\frac{\rho R^3}{3\varepsilon_0}\left(\frac{1}{z^2} - \frac{1/64}{(z-R/2)^2}\right) \\ -R < z < \frac{R}{4}: & \frac{\rho z}{3\varepsilon_0} + \frac{\rho R^3}{3\varepsilon_0 \cdot 64 \cdot (z-R/2)^2} \\ \frac{R}{4} < z < \frac{3R}{4}: & \frac{\rho z}{3\varepsilon_0} - \frac{\rho(z-R/2)}{3\cdot \varepsilon_0} = \frac{\rho R}{6\varepsilon_0} \leftarrow \text{constant!} \\ \frac{3R}{4} < z < R: & \frac{\rho z}{3\varepsilon_0} - \frac{\rho R^3}{3\varepsilon_0 \cdot 64 \cdot (z-R/2)^2} \\ z > R: & \frac{\rho R^3}{3\varepsilon_0}\left(\frac{1}{z^2} - \frac{1/64}{(z-R/2)^2}\right) \end{cases}$$

Along **X** axis:



Both $E_x \neq 0$, $E_z \neq 0$

$E_y = 0$.



$$E_x^{(x)} = \begin{cases} x < R: & \dfrac{\rho \cdot x}{3\varepsilon_0} - \dfrac{Q_0/64}{4\pi\varepsilon_0} \dfrac{1}{x^2 + R^2/4} \cdot \dfrac{x}{\sqrt{x^2 + R^2/4}} \\[2ex] x > R: & \dfrac{Q_0}{4\pi\varepsilon_0} \dfrac{1}{x^2} - \dfrac{Q_0/64}{4\pi\varepsilon_0} \dfrac{x}{\left(x^2 + R^2/4\right)^{3/2}} \end{cases}$$

$$\boxed{E_z^{(x)} = - \frac{Q_0/64}{4\pi\varepsilon_0} \frac{R/2}{\left(x^2 + R^2/4\right)^{3/2}}}$$