Rutgers Physics 385: Electromagnetism I (Fall'15 / Gershtein)
Final Exam - Dec 15, 2015

This is a closed book/notes exam. A one-sided 8.5x11 sheet with only formulae is allowed. Please attach the sheet to your solutions. Calculators are not needed. Indicate explicitly which problems should be graded. Indicate your reasoning clearly and step-by-step. Exam duration - 12 to 3 PM.

Useful Formulae

derivatives in spherical coordinates (r, theta, phi):

$$\vec{\nabla} f = \frac{\partial f}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial f}{\partial \theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial f}{\partial \phi}\hat{\phi}$$

$$\vec{\nabla}\cdot\vec{A} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 A_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(A_\theta\sin\theta) + \frac{1}{r\sin\theta}\frac{\partial A_\phi}{\partial\phi}$$

$$\vec{\nabla}\times\vec{A} = \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}(A_\phi\sin\theta) - \frac{\partial A_\theta}{\partial\phi}\right]\hat{r} + \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial A_r}{\partial\phi} - \frac{\partial}{\partial r}(rA_\phi)\right]\hat{\theta} + \frac{1}{r}\left[\frac{\partial}{\partial r}(rA_\theta) - \frac{\partial A_r}{\partial\theta}\right]\hat{\phi}$$

derivatives in cylindrical coordinates (r, phi, z):

$$\vec{\nabla} f = \frac{\partial f}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial f}{\partial\phi}\hat{\phi} + \frac{\partial f}{\partial z}\hat{z}$$

$$\vec{\nabla}\cdot\vec{A} = \frac{1}{r}\frac{\partial}{\partial r}(rA_r) + \frac{1}{r}\frac{\partial A_\phi}{\partial\phi} + \frac{\partial A_z}{\partial z}$$

$$\vec{\nabla}\times\vec{A} = [\frac{1}{r}\frac{\partial A_z}{\partial\phi} - \frac{\partial A_\phi}{\partial z}]\hat{r} + [\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r}]\hat{\phi} + \frac{1}{r}[\frac{\partial}{\partial r}(rA_\phi) - \frac{\partial A_r}{\partial\phi}]\hat{k}$$

Legandre polynomials:
$P_0(x) = 1$
$P_1(x) = x$
$P_2(x) = (3x^2-1)/2$
$P_3(x) = (5x^3-3x)/2$

General Laplace equation solution in cylindrical coordinates if separation of variables is possible:

$$V(\rho,\phi) = A_0\ln\rho + B_0 + \sum_{n=1}^{\infty}[\rho^n(A_n\cos n\phi + B_n\sin n\phi) + \frac{1}{\rho^n}(C_n\cos n\phi + D_n\sin n\phi)]$$

Section A - Do any 3 out of 4 problems (4 pts each, 12 total for the section)

A1. An infinite cylinder of radius $\boldsymbol{R}$ carries volume charge density $\boldsymbol{\rho = kr}$. Find electric field inside and outside the cylinder.

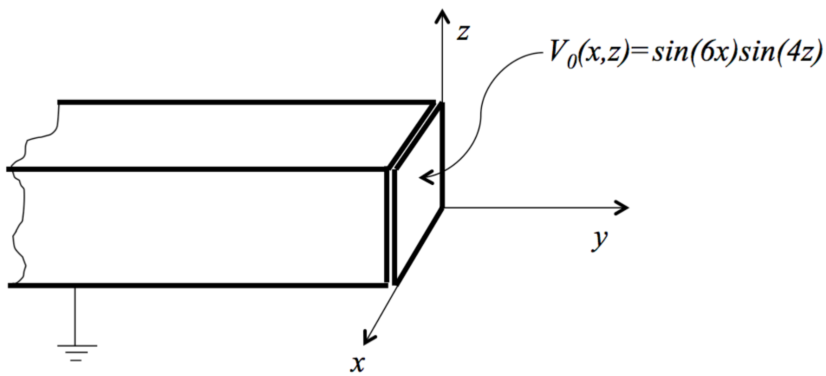
A2. Find energy stored in a spherical capacitor, with plate radii equal to $\boldsymbol{a}$ and $\boldsymbol{b}$, charged to voltage $\boldsymbol{V}$.

A3. Two infinitely long wires run parallel to each other with separation d, carrying equal currents I. Find the force per unit length on each of the wires.

A4. Magnetic vector potential is given in cylindrical coordinates as $\vec{A} = k\hat{\phi}$, where k is a constant. What current density would create this field?

Section B - Do any 2 out of 3 problems (7 pts each, 14 total for the section)

B1. A long square tube (side equal to π) consists of four metal sides which are welded together and grounded (see figure below). The tube is topped by a plate, insulated from the tube, held at a potential $V_0(x,z)=\sin(6x)\sin(4z)$. Find the potential inside the tube.



B2. The potential of the surface of a hollow empty sphere of radius R is given by $V(\theta) = C \sin^2(\theta/2)$, where θ is the polar angle and C is some constant. Find the potential inside the sphere.

B3. A sphere of radius R carries a polarization $\vec{P} = kr\hat{r}$, where k is a constant. Calculate the bound charges and the fields $\vec{E}$ and $\vec{D}$ everywhere.

Section C - Do any 2 out of 3 problems (7 pts each, 14 total for the section)

C1. A uniform surface current $\vec{K} = K\hat{x}$ flows along the *xy* plane. An infinite slab of material with magnetic susceptibility χ_m rests on top of the *xy* plane, occupying $0 < z < d$. Find bound currents, magnetization, and B and H fields everywhere.

C2. A vinyl record of radius R carries uniform surface charge density σ and is rotating with angular speed ω. Find it's magnetic dipole moment.

C3. An infinitely long cylinder or radius R carries a volume charge density ρ and is rotating with angular speed ω along its axis. Find the magnetic filed inside and outside of the cylinder.