16.15

a) $\vec{\nabla} f(\hat{n}\cdot\vec{r} - ct)$

$$\frac{\partial}{\partial x} f(\hat{n}\cdot\vec{r}-ct) = f'(\hat{n}\cdot\vec{r}-ct)\cdot n_x$$, similarly for y and z

$$\vec{\nabla} f(\hat{n}\times\vec{r}-ct) = \hat{n}\cdot f'(\hat{n}\cdot\vec{r}-ct)$$

b) 3-D wave equ: $\frac{\partial^2}{\partial t^2} f(\vec{r},t) = c^2 \nabla^2 f(\vec{r},t)$

$$\frac{\partial^2}{\partial t^2} f(\hat{n}\cdot\vec{r}-ct) = c^2 f''(\hat{u}\cdot\vec{r}-ct)$$

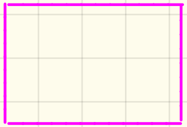
$$\nabla^2 f(\vec{r},t) = \vec{\nabla}\left(\vec{\nabla}\cdot f(\hat{n}\vec{r}-ct)\right) = \vec{\nabla}\left(\hat{n} f'(\hat{n}\vec{r}-ct)\right) =$$

$$= \hat{n}\cdot\vec{\nabla} f'(\hat{n}\vec{r}-ct) = \hat{n}\cdot\hat{n}\cdot f''(\hat{n}\vec{r}-ct) = f''(\hat{n}r-ct)$$

c) let's pick x direction along $\hat{n}$

$$f(\hat{n}\cdot\vec{r}-ct) = f(x-ct) \longleftarrow$$ does not depend on y, z
propagates toward +x at speed c.

16.17



$$x \to x + u(x,t)$$

a)

$$x + u(x,t) \qquad x + dx + u(x+dx, t) =$$

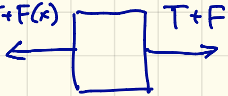
$$= x + dx + \frac{\partial u}{\partial x} \cdot dx + u(x,t)$$

length dx unstretched $\to$ length $dx + \frac{\partial u}{\partial x} dx$

$$\frac{\Delta F}{A} = YM \frac{\Delta x}{x} = YM \cdot \frac{\partial u}{\partial x}$$

extra tension in the string: $\Delta F = YM \cdot A \cdot \frac{\partial u}{\partial x}$

b) $-T + F(x)$ $\qquad$ $T + F(x+dx)$



$$\rho \cdot A\, dx \cdot \frac{\partial^2 u}{\partial t^2} = F(x+dx) - F(x) = YM \cdot A \cdot \frac{\partial^2 u}{\partial x^2} \cdot dx$$

$$\frac{\partial^2 u}{\partial t^2} = \frac{YM}{\rho} \cdot \frac{\partial^2 u}{\partial x^2} \implies c = \sqrt{\frac{YM}{\rho}}$$

16.20

$$\overleftrightarrow{\Sigma} = \begin{bmatrix} xz & z^2 & 0 \\ z^2 & 0 & -y \\ 0 & -y & 0 \end{bmatrix}$$

Surface: $x^2 + y^2 + 2z^2 = 4$ → ellipsoid

Point $(1,1,1)$ $\quad f(x,y,z) = x^2 + y^2 + 2z^2 - 4$, $\hat{n} = \dfrac{\vec{\nabla} f}{|\vec{\nabla} f|}$

$$\frac{\partial f}{\partial x} = 2x \quad \frac{\partial f}{\partial y} = 2y \quad \frac{\partial f}{\partial z} = 4z$$

$$\vec{\nabla} f(1,1,1) = (2,2,4)$$

$$|\vec{\nabla} f(1,1,1)| = \sqrt{4+4+16} = \sqrt{24}$$

$$\overleftrightarrow{\Sigma} \cdot \hat{n} = \frac{1}{\sqrt{24}} \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & -1 \\ 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ 2 \\ 4 \end{bmatrix} = \frac{1}{\sqrt{24}} \begin{bmatrix} 4 \\ -2 \\ -2 \end{bmatrix} ; \quad d\vec{F} = \frac{dA}{\sqrt{24}} \begin{bmatrix} 4 \\ -2 \\ -2 \end{bmatrix}$$

16.24

$$\vec{u}(\vec{r}) = \vec{\Theta} \times \vec{r} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \Theta_x & \Theta_y & \Theta_z \\ x & y & z \end{vmatrix} = \begin{bmatrix} z\Theta_y - y\Theta_z \\ x\Theta_z - z\Theta_x \\ y\Theta_x - x\Theta_y \end{bmatrix} = \begin{bmatrix} u_x \\ u_y \\ u_z \end{bmatrix}$$

$$\overleftrightarrow{D} = \begin{bmatrix} \frac{\partial u_x}{\partial x} & \frac{\partial u_x}{\partial y} & \frac{\partial u_x}{\partial z} \\ \frac{\partial u_y}{\partial x} & \frac{\partial u_y}{\partial y} & \frac{\partial u_y}{\partial z} \\ \frac{\partial u_z}{\partial x} & \frac{\partial u_z}{\partial y} & \frac{\partial u_z}{\partial z} \end{bmatrix} = \begin{bmatrix} 0 & -\Theta_z & \Theta_y \\ \Theta_z & 0 & -\Theta_x \\ -\Theta_y & \Theta_x & 0 \end{bmatrix}, \quad \text{q.e.d.}$$

16.27 YM derivation:

$$\overleftrightarrow{\Sigma} = \begin{pmatrix} \sigma & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \leftarrow \text{stress along } x\text{, no shears}$$

a)
$$\overleftrightarrow{E} = \frac{3\alpha \overleftrightarrow{\Sigma} - (\alpha-\beta)\,\mathrm{tr}\,\overleftrightarrow{\Sigma}\,\overleftrightarrow{1}}{3\alpha\beta} = \frac{3\alpha}{3\alpha\beta}\begin{pmatrix} \sigma & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} - \frac{(\alpha-\beta)\cdot\sigma}{3\alpha\beta}\cdot\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

stretch

compress

$$\begin{pmatrix} \frac{2\alpha+\beta}{3\alpha\beta}\sigma & 0 & 0 \\ 0 & -\frac{(\alpha-\beta)}{3\alpha\beta}\sigma & 0 \\ 0 & 0 & -\frac{(\alpha-\beta)}{3\alpha\beta}\sigma \end{pmatrix}$$

b)

$$\frac{\Delta F}{\Delta A} = YM \cdot \frac{\Delta l}{l}$$

$$\overset{\shortparallel}{\sigma} \qquad \overset{\shortparallel}{E_{11}}$$

$3 \cdot BM = \alpha$

$2\, SM = \beta$

c)

$$YM = \frac{\sigma}{\frac{(2\alpha+\beta)\,\sigma}{3\alpha\beta}} = \frac{3\alpha\beta}{2\alpha+\beta} = \frac{3 \cdot 3 \cdot \cancel{2} \cdot BM \cdot SM}{3 \cdot \cancel{2} BM + \cancel{2} SM}$$

$$\boxed{YM = \frac{9 \cdot BM \cdot SM}{3 \cdot BM + SM}}$$

16.28

a) Poisson ratio: $\dfrac{\text{compression along } y}{\text{stretch along } x} = -\dfrac{E_{22}}{E_{11}}$

b) $\nu = -\dfrac{E_{22}}{E_{11}} = \dfrac{\alpha-\beta}{3\alpha\beta} \cdot \dfrac{3\alpha\beta}{2\alpha+\beta} = \dfrac{\alpha-\beta}{2\alpha+\beta}$

c) $\nu = \dfrac{3BM - 2SM}{6BM + 2SM}$

d)

iron	0.31
steel	0.26
stone	0.34
water	0.5

if $SM \ll BM$ $\nu \sim 0.5$

(16,31)

$\Delta t = 12\,\text{min}$

$c_L = 5.25 \text{ km/s}$

$c_T = 3.0 \text{ km/s}$

$$t_L = \frac{d}{c_L} \qquad t_T = \frac{d}{c_T}$$

$$\Delta t = d\left(\frac{1}{c_T} - \frac{1}{c_L}\right)$$

$$d = \frac{c_L c_T}{c_L - c_T} \cdot \Delta t \approx 5000 \text{ km}$$

16,33

$$\rho \frac{d\vec{v}}{dt} = \rho\vec{g} - \vec{\nabla} p$$

if $\vec{v} = 0$, $\rho\vec{g} = \vec{\nabla} p$

line integral from point 1 to 2:

$$\int_1^2 \rho\vec{g}\, d\vec{r} = \int_1^2 \vec{\nabla} p\, d\vec{r}$$

$$= \int_1^2 \left(\frac{\partial p}{\partial x} \cdot dx + \frac{\partial p}{\partial y} dy + \frac{\partial p}{\partial z} dz \right) = \int_1^2 dp = p_2 - p_1$$

$$= \rho g (z_2 - z_1)$$

$$p_2 - p_1 = \rho g (z_2 - z_1) \quad \text{q.e.d.}$$