15.2

$A + B \to C + D$

Galileo transform:

$$m_A \vec{v}_A + m_B \vec{v}_B = m_C \vec{v}_C + m_D \vec{v}_D$$

$$\vec{v} = \vec{v}' + \vec{V}$$

$$0 = \underbrace{m_A \vec{v}_A{}' + m_B \vec{v}_B{}' - m_C \vec{v}_C{}' - m_D \vec{v}_D{}'}_{=0 \text{ if momentum conserves in } K'} + (m_A + m_B - m_C - m_D)\vec{V}$$

Since $\vec{V}$ is arbitrary,

$$m_A + m_B - m_C - m_D = 0$$

$$m_A + m_B = m_C + m_D$$

mass conservation!

15.3

$v = 8 \cdot 10^{3}$ m/s $\qquad \frac{v}{c} = 2.66 \cdot 10^{-5}$

$c = 3 \cdot 10^{8}$ m/s

$$\gamma = \frac{1}{\left(1 - \frac{v^2}{c^2}\right)^{1/2}} \approx 1 + \frac{v^2}{2c^2} = 1 + 3.6 \cdot 10^{-10}$$

$$3.6 \cdot 10^{-10} \cdot 3.6 \cdot 10^{3}\, s \simeq 13 \cdot 10^{-7} = 1.3 \cdot 10^{-6}\, s = 1{,}3\, \mu s.$$

clock on the satellite would show
1 hour $- 1{,}3\,\mu s$.

15.5

$\beta = 0.95: \quad \gamma = \frac{1}{\sqrt{1-0.95^2}} \simeq 3.2$ ← same for trips there and back

$\Delta t = 80$ years

$\Delta t_{there} = 40$ years $\qquad \Delta t'_{there} = \frac{\Delta t_{there}}{\gamma}$

$\Delta t_{back} = 40$ years $\qquad \Delta t'_{back} = \frac{\Delta t_{back}}{\gamma}$

$\Delta t' = \frac{\Delta t}{\gamma} \simeq 25$ years ← clocks on A say that's how long the trip took

→ twin A is $80 - 25 = 55$ years younger.

15.7

$\tau = 1.5\,\mu s$ (proper) $\quad h = 2 \cdot 10^3\, m$

$v = 0.99c \rightarrow \gamma = 7.1$

$\tau_{earth} = \gamma \tau \leftarrow$ moving μ live longer

time the Earth

$$t = \frac{h}{v} \qquad N = N_0 \cdot 2^{-t/\tau}$$

$$N_{classicle} = N_0 \cdot 2^{-\frac{h}{v\gamma\tau}} = 650 \cdot 2^{-0.63} = \underline{\underline{420}}$$

$$N_{true} = N_0 \cdot 2^{-\frac{h}{v\tau}} = 650 \cdot 2^{-4.5} = \underline{\underline{29}}$$

15.8 $\tau = 1.8 \cdot 10^{-8}\,s$

a) $\tau(0.8c) = \tau / \sqrt{1-0.8^2} = \tau \cdot \frac{1}{0.6} \quad 3 \cdot 10^{-8}\,s$

b) time of flight

$t = \frac{d}{0.8c}$ $\qquad N = N_0 \cdot 2^{-\frac{d}{0.8c}\frac{0.6}{\tau}} = 32000 \cdot 2^{-5} \approx 1000$

c)

$N_{classical} = N_0 \cdot 2^{-\frac{d}{0.8c}\frac{1}{\tau}} \approx 32{,}000 \cdot 2^{-8.3} \approx 100$