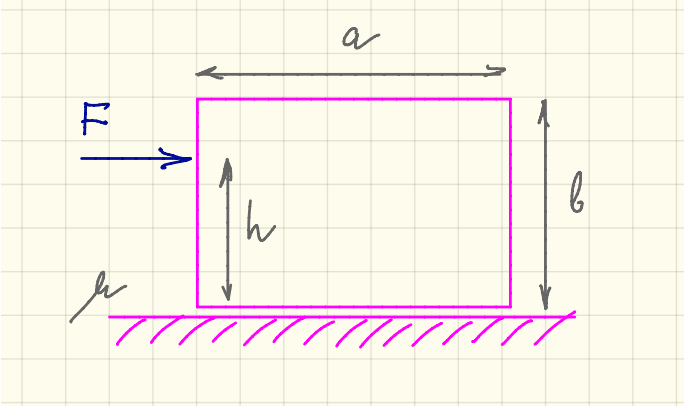
Rutgers Physics 381 Mechanics I (Fall'16/Gershtein)

Final Exam – December 19, 2016

This is a closed book/notes exam. A one-sided 8.5x11 sheet with only formulae is allowed. Please attach the sheet to your solutions. Calculators are not needed. Exam duration – 3 hours.

1. A block of height ***b*** and width ***a*** is on a rough surface, with friction coefficient μ. A horizontal force is applied at height ***h***, as shown in the figure below. The force is slowly increasing. What happens to the block first: does it slide or topple?



2. A particle of mass ***m*** can move along *x* in a potential $U(x)= -a^2 x^2 + \frac{1}{2} b^2 x^4$.

a) find equilibrium locations and specify which are stable

b) determine frequency of small oscillations around the stable equilibrium.

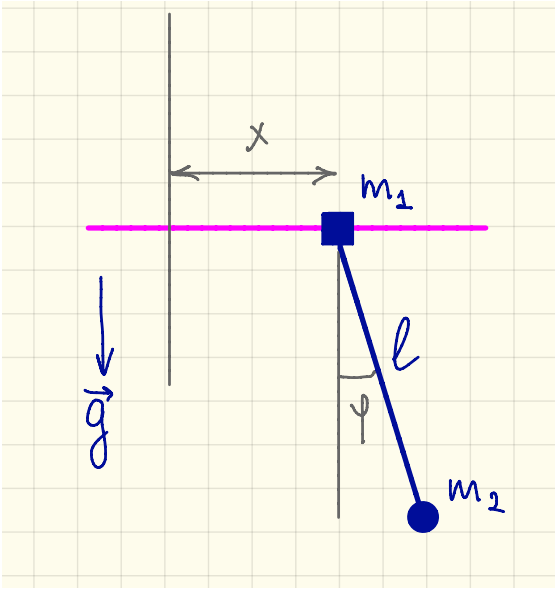
3. A particle of mass ***m*** moving with speed v_0 encounters media in which it experiences drag force $F= -c\, v^{3/2}$, where ***c*** is a constant.

a) find speed of the particle as a function of time. Does it ever come to rest?

b) will the particle have a finite range?

If yes, calculate it, if not, prove that it does not exist

4. A mass m_1 can move without friction on a horizontal rail. A weightless rod of length l is attached to it with a hinge, and mass m_2 is attached to its other end (see figure below). Write down the Langrangian of the system in uniform gravitational field. Which of the two coordinates is ignorable? What conservation law does it correspond to?



5. For the system above, find the frequency of small oscillations of the system. *(Hint: first Taylor-expand the Lagrangian keeping terms up to the second power of x, ϕ, and their time derivatives, then write down the Lagrange equations and solve them)*

6. A spherical planet of radius R and mass M is covered by ocean. The planet rotates around its axis with angular speed Ω. That rotation makes the ocean bulge around equator. Assuming that the depth of the ocean is much smaller then the radius of the planet, find the difference in ocean depth at the equator and the poles. *(hint: recall the ocean surface should be an equipotential surface)*